Stochastic Resonance in Retinomorphic Neural Networks with Nonidentical Photoreceptors and Noisy McCulloch-Pitts Neurons

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Abstract—We propose a simple retinomorphic neural network that consists of photoreceptors generating nonuniform outputs for common optical inputs with random offsets, an ensemble of noisy McCulloch-Pitts neurons each of which has random threshold values, local synaptic connections between the photoreceptors and the neurons with variable receptive fields (RFs), output cells, and local synaptic connections between the neurons and output cells. Through numerical simulations, we observed stochastic resonance among the proposed pixels. We calculated correlation values between the optical inputs and the outputs as a function of the RF size and intensities of the random components in photoreceptors and the McCulloch-Pitts neurons, and then found nonzero optimal RF sizes as well as optimal noise intensities of the neurons under the nonidentical photoreceptors. This implies that SR-based night-scope image sensors with an array of nonidentical photosensors would be developed with less efforts to implement uniform pixel devices.

1. Introduction

Stochastic resonance (SR) has recently been spotlighted in the field of electrical engineering, which is motivated by a wide variety of sensing applications to detect weak signals [1]. Among the SR applications, night-scope (dark) image sensing [2] seems to particularly be promising in semiconductor research fields because present image sensors employ very precise (and thus expensive) devices for photon sensing or counting. However, SR would be useful only for detecting dark light in a single photosensor, but not for canceling mismatches between the photosensors in the sensor array (image sensor). In this paper, we propose a retinomorphic neural network utilizing SR to cancel the pixel mismatches while detecting dark images.

Recently, Funke *et al.* reported that a visual pathway in a cat primary visual cortex optimally utilized a SR-like process to improve signal detection while preventing spurious noise-induced activity and keeping the SNR high [3]. Although the mechanism is still unclear, one may assume that i) SR without optimal tuning of noise intensities [4] underlies the fundamental mechanism and ii) the visual pathway from photoreceptors to cortical neurons may cause extremely large receptive field (RF). Under these assumptions, we here consider a relatively short pathway, i.e., from
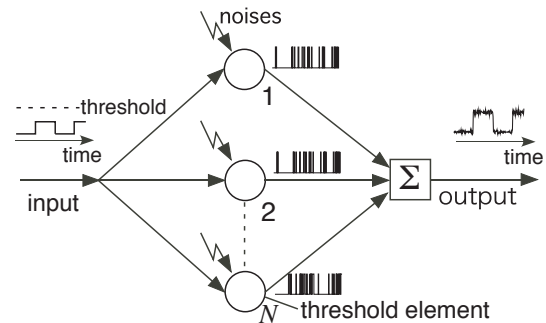
(a) Summing network of threshold elements
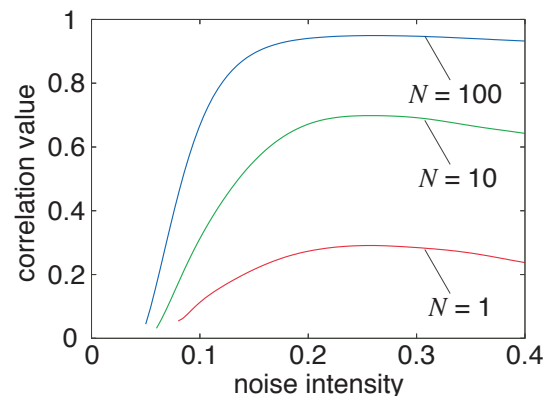
(b) Correlation values between input and output signals vs noise intensity

Figure 1: Stochastic resonance (SR) without optimal tuning of noise intensity [4]

photoreceptors to the subsequent layer in vertebrate retinae, and propose a retinomorphic neural network which has parallel SR components with variable RFs.

2. Brief Review of SR Models

Figure 1 shows a basic SR model proposed in [4]. A subthreshold input is commonly given to N threshold elements, as illustrated in Fig. 1(a). FitzHuge-Nagumo neurons were used in the original paper, but one may use McCulloch-Pitts type neurons instead without loss of generality. Each neuron accepts external uncorrelated noises,
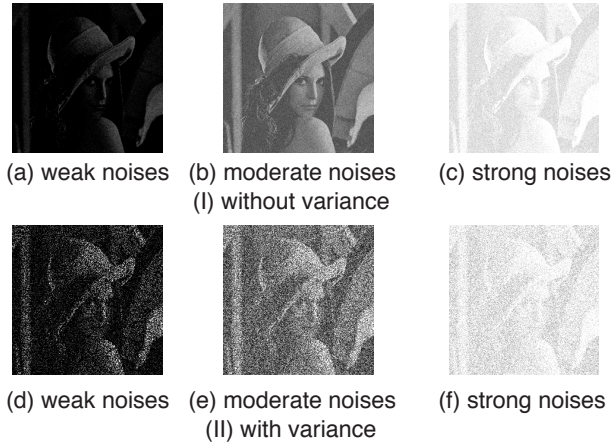
Figure 2: SR in array of independent pixels

which leads the neurons to fire with high (or low) possibility when the subthreshold input is high (or low). When the outputs were summed, the uncorrelated noises tend to be cancelled each other as N increases. Examples with McCulloch-Pitts type neurons were shown in Fig. 1(b) where correlations values between the input and output signals were plotted as a function of the noise intensity. One can observe that the correlation values are increased as N increases and the values tend to be insensitive to noise intensity (> 0.2 when $N = 100$). This means, when a large number of neurons are used in the system, the subthreshold (weak) input can be detected without optimal tuning of the noise intensity [4].

The concept of SR has been expanded to dark-image sensing applications [2]. Let us consider a 2-D array of pixels, and assume that each pixel consists of the same SR model in Fig. 1(a). The array accepts dark (subthreshold) images, and thus the array's output would be always zero when external noises were not given. As the noise intensity increases, nonzero outputs appeared, as shown in Figs. 2(a) to (c). When each pixel has random offset values, they are directly detected through the SR process, as shown in Figs. 2(d) to (f). Such a random offset is generally observed in photodiodes as dark currents. Consequently, SR would be useful for sensing dark images, however, the random offsets would also be detected in practical systems.

3. Retinomorphic SR Model

We can consider three types of SR networks for nightscope image sensors from the viewpoint of the practical hardware implementation. The first structure is illustrated in Fig. 3(a) where an optical input to a pixel is given to a single McCulloch-Pitts neuron. Each neuron accepts temporal noises, and the temporal average of the neuron represents the pixel output. With this setup, correlation values between the input and the output would be low because the single pixel exactly corresponds to the network of $N = 1$ in
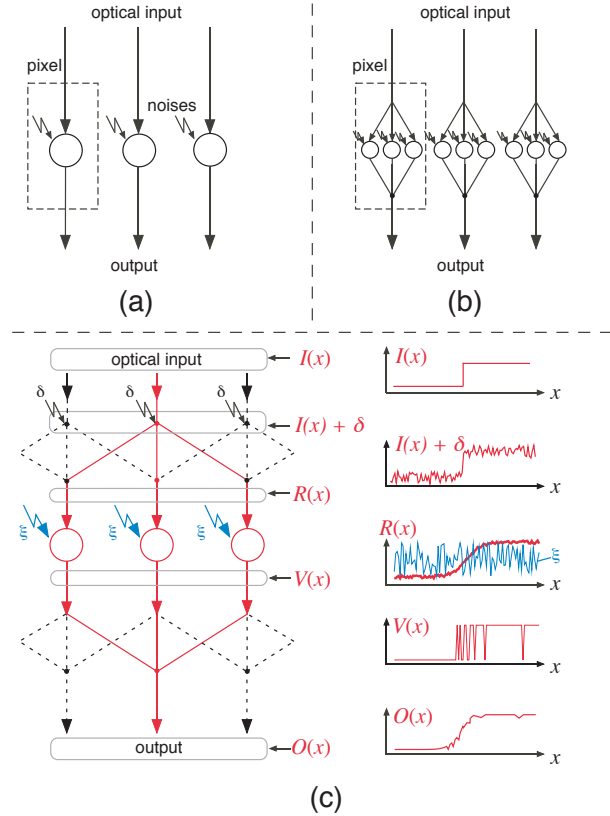

Figure 3: Three types of SR models for possible hardware implementation

Fig. 1. To increase the correlation value, one can employ the second structure shown in Fig. 3(b). This setup certainly increases the correlation values, however, the integration density would significantly be decreased due to the large number of threshold components per pixel. Therefore we propose a superimposed structure aiming at high integration density while keeping the correlation values high. Let us consider a 1-D network as shown in Fig. 3(c) where the optical input distribution is represented by $I(x)$. Output distribution of photoreceptors are thus defined by $I(x)+\delta(x)$ where $\delta(x)$ represents the spatial random noise (pixel variations) given by $m \cdot N(0, 1)$ [$N(0, 1)$ is the Gaussian noise with zero mean and unity standard deviation]. Inputs to neurons via local synaptic connections between photoreceptors and neurons were then defined by

$$R(x) = \int \big(I(x) + \delta(x)\big) \cdot g(x - X)\, dX, \qquad (1)$$

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{x^2}{2\sigma^2}\right],$$

where σ represents the RF size. Output distribution of the neurons is thus

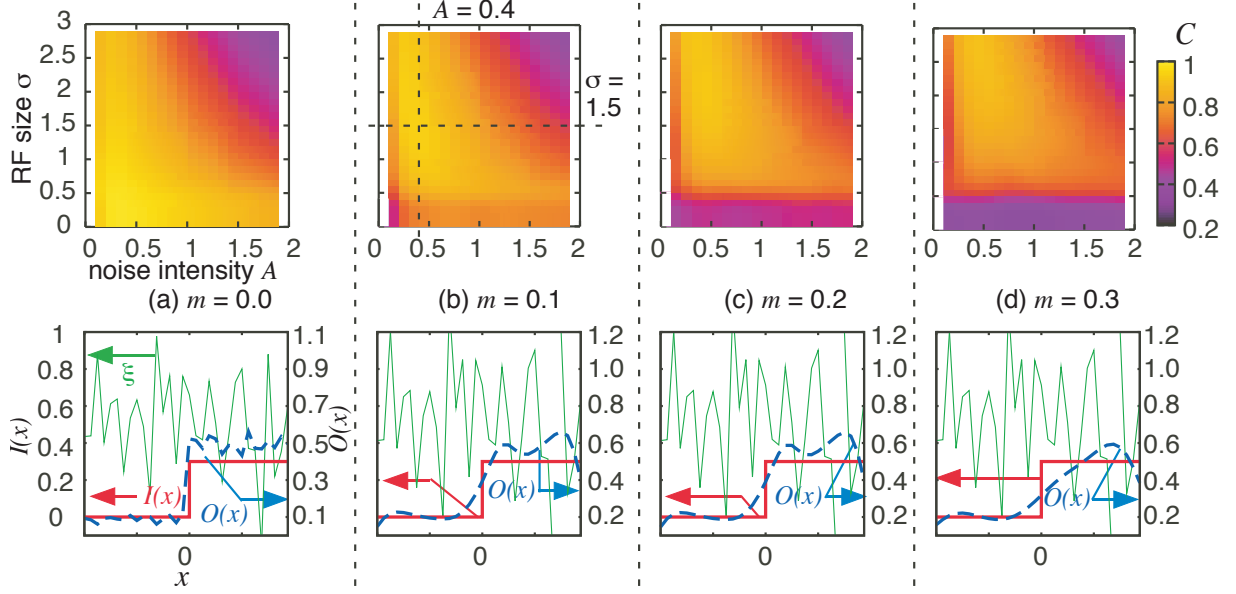$$V(x) = H\big(R(x) - \xi(t)\big), \qquad (2)$$

Figure 4: Simulation results of proposed 1-D network for step optical inputs
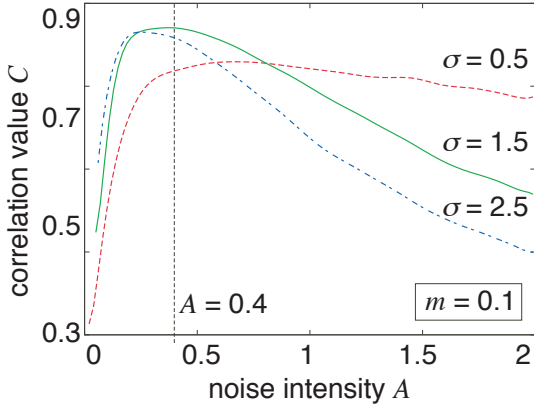

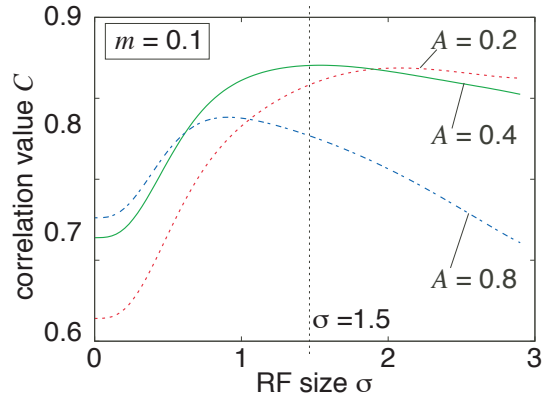Figure 5: Horizontal cross sections of Fig. 4(b).

Figure 6: Vertical cross sections of Fig. 4(b).

where $H(\cdot)$ represents a step function, and $\xi(t)$ the temporal random noise given by $A \cdot N(0,1) + \theta$ (A: standard deviation, θ: mean threshold). The final output via local synaptic connections between the neurons and output cells is then given by

$$O(x) = \int V(x) \cdot g(x - X)\, dX. \qquad (3)$$

With this model, we examine SR behaviors by changing m (spatial randomness), σ (RF size), and A (temporal randomness being necessary for SR).

4. Results

In the following simulations, we assume $\theta = 0.5$ and $I(x) = H(x)$. Figure 4 (top) shows density plots of correlation values between the optical input $I(x)$ and final output

$O(x)$ as a function of σ and A with four different values of ms. Figure 4 (bottom) shows distributions of $I(x)$, $O(x)$, and random threshold $\xi(t)$ at arbitrary t. If photoreceptors are identical ($m = 0$, Fig. 4(a)), the maximum correlation value was obtained when the RF size (σ) was zero, as expected. However, if photoreceptors are not identical ($m = 0.1$, Fig. 4(b)), the correlation peak was moved to nonzero σ while keeping the peak value high. This is unexpected and surprising result implying that a nonzero RF size would be necessary for SR among nonidentical components. As m increases (Fig. 4(c) and (d)), the peak shifted to higher σ, however, the peak values were decayed slowly.

Figure 5 shows horizontal cross sections of Fig. 4(b). As in the basic SR network (Fig. 1), the correlation values had a peak for variable noise intensity (A). It should be no-
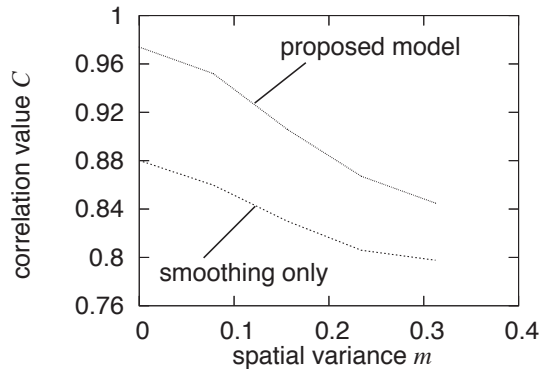
Figure 7: Maximum correlation values vs spatial variance

Figure 8: 2-D simulation results

ticed that the correlation values did not decrease suddenly as noise intensity A increased. When $\sigma = 1.5$, for example, the peak value was almost insensitive within $0.2 < A < 0.5$. Since σ qualitatively represents the number of neurons in the RF of each pixel, increasing σ may improve the correlation as in the basic SR network. However, nonzero σ causes smoothing of the optical input. Therefore there may exist upper bounds of σ. Figure 6 shows the vertical cross sections of Fig. 4(b) representing correlation values as a function of σ. The peak value was obtained around $\sigma \approx 1.5$ and $A = 0.4$, which proves that nonzero σ (RF size) is necessary for obtaining higher correlation values in this SR system with nonidentical pixels ($m > 0$).

To prove the effects of RFs in our SR model to improve the SNR, we calculated peak correlation values in comparison with just a smoothing model that has the same structure as Fig. 3(c) assuming zero RF size between $V(x)$ and $O(x)$ layers. Figure 7 plots the peak values as a function of spatial variance m. In this case, $I(t)$ was set to 0.2 when $t > 0$ and 0 when $t \leq 0$. For given m, the peak values were scanned by sweeping two parameters A (noise intensity) and σ (RF size). The correlation values of the proposed model was always larger than that of the smoothing model, and the difference slightly decreased as m increased.

Finally, we evaluated performances of a 2-D network with two distinct RF sizes ($\sigma = 0.3, 1.5$). Figure 8 show the results ($A = 0.4$, $m = 0.1$, and $\theta = 0.5$). Binary input image $I(x, y)$ is shown in Fig. 8(a), whereas density plots of outputs of neurons $V(x, y)$ [(b) and (c)] and final output cells $O(x, y)$ [(d) and (e)] with different RF sizes are shown. By comparing the final outputs of small RF ($\sigma = 0.3$) and relatively large RF ($\sigma = 1.5$), we conclude that expanding RF sizes are useful for obtaining visually-better output image through SR among nonidentical pixels. The important thing here is that the underlying mechanism of the performance increase mainly resulted from SR without tuning rather than just any image smoothing, as proved in Fig. 7.
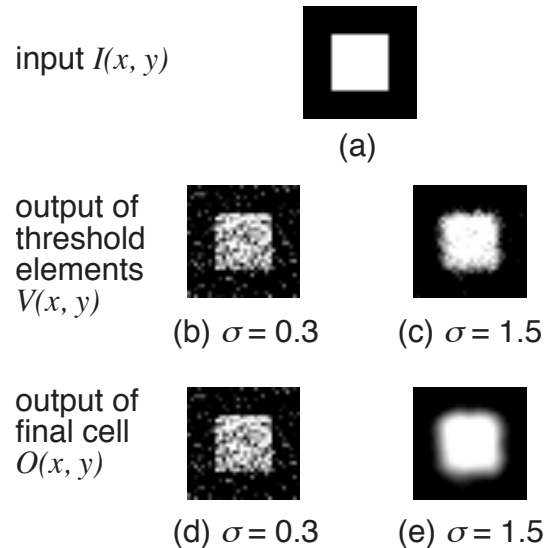
5. Summary

We proposed a simple retinomorphic neural network consisting of nonidentical photoreceptors towards the development of high-density night-scope image sensor systems. Through numerical simulations, we observed a new class of stochastic resonance among the nonidentical pixels. We calculated correlation values between the optical inputs and the output as a function of the receptive-field (RF) size and intensities of the random components in photoreceptors and the McCulloch-Pitts neurons, and then found nonzero optimal sizes of the RF as well as optimal noise intensities of the neurons under the nonidentical photoreceptors.

References

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