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Wolframのルール90/150に基づく履歴参照型アナログセルオートマトンとそのハードウェア化

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Circuit Implementation of Historic Analog Cellular Automata based on Wolfram's Rule 90/150

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Standard cellular automata (CA) employes transition rules where a cell's state is updated in terms of a previous state (1 step before) of the cell's neighbors. This standard framework has recently been expanded by introducing historic memory capabilities into CAs, *i.e.*, a cell's state is updated not only the neighbor's previous state, but also the past states (2, 3, $\cdots$, M steps before), which resulted in generating statistically good random numbers [1], spatialized prisoner's dilemma [2], multi-fractal properties of Wolfram's rule 90 [3], and so on.

We here propose semi-analog CAs based on Wolfram's rules 90 and 150 having analog historic memory capabilities. Original difference equations of 1-D CAs with rules 90 and 150 are given by $x_i^{t+1} = f(x_{i-1}^t, x_i^t, x_{i+1}^t)$ where x_i^t represents the i-th cells state ($\in 0, 1$) at time t, $f(\cdot)$ the exclusive OR function defined by $x_{i-1}^t \oplus x_{i+1}^t$ and $x_{i-1}^t \oplus x_i^t \oplus x_{i+1}^t$ for rules 90 and 150, respectively. We rewrote the binary difference equation into analog difference equations to include historic memory terms. Let us start from a leaky integrator model given by $\dot{x}_i = -\alpha x_i + f(\cdot)$ ($\alpha \geq 0$) that can be approximated as $x_i^{t+\Delta t} = (1 - \Delta t \alpha) x_i^t + f(\cdot)$ ($\Delta t \ll 1$). When $\Delta t = 1$, we obtain

$$x_i^{(t+1)} = (1 - \alpha) x_i^t + f(x_{i-1}^t, x_i^t, x_{i+1}^t), \tag{1}$$

which represents our analog CA dynamics with analog historic memories. When $\alpha = 1$ the equation above exactly equals to the original difference equation, whereas for $\alpha < 1$, the historic memory effect appears, *i.e.*, the past cell's states are preserved by the leaky integrator as analog values. Note that the activate functions $f(\cdot)$ must also be expanded to continuous analog functions because of this analog expansion.

Numerical simulation results showed that i) the semi-analog CA generated self-similar patterns as conventional CAs when the historic memory effect was attenuated, whereas ii) dense spatial patterns were generated while maintaining the self-similar properties. Moreover we designed analog electrical circuits for CMOS LSIs implementing the proposed historic analog CAs, aiming at the applications to random-number generation. We demonstrate the operations by both circuit simulations (SPICE) and experiments using discrete semiconductor devices.

[1] Alonso-Sanz, R., Bull, L. (2008). Random number generation by cellular automata with memory. Int. J. Modern Physics C, 19(2), 351-367.
[2] Alonso-Sanz, R., Martin, M. (2005). The Spatialized Prisoner's Dilemma with limited trailing memory. In Modeling Cooperative Behaviour In the Social Sciences: Eigth Granada Lectures. P.L.Garrido et al. (eds.). AIP Conf. Proc. 779, 124-127.
[3] Sanchez, J.R., Alonso-Sanz, R. (2004). Multifractal Properties of R90 Cellular Automaton with Memory. Int.J. Modern Physics C, 15(10), 1461-1470.