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Topic: Pattern Formation — Preference: Poster

Spatial Pattern Formation of Diffusive Lotka-Volterra System on Analog Integrated Circuits

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Background: In [1, 2], the authors fabricated analog electrical circuits for a three-variable Lotka-Volterra (LV) system [3] that exhibits chaotic oscillations. They can easily be implemented on silicon chip because of their compact structure and low-power consumption, which implies that a large-scale 2D array of the LV system can be implemented on LSIs. Therefore, in this report, we examine spatiotemporal properties on the array circuit to explore possible applications of the LSI.
Method: Dynamics of the LV circuit were formulated by a linear combination of exponential functions [1], while that of diffusive-coupled LV circuits are given by

$$\begin{aligned}
C\frac{\partial V_1}{\partial t} &= G_1\nabla^2 V_1 + I_1 - I_0^{(\mathrm{M}1)}e^{\frac{\kappa}{V_T}V_1} - I_0^{(\mathrm{M}c)}e^{\frac{\kappa}{V_T}V_2} - I_0^{(\mathrm{M}k)}e^{\frac{\kappa}{V_T}V_3}\\
C\frac{\partial V_2}{\partial t} &= G_2\nabla^2 V_2 + I_2 - I_0^{(\mathrm{M}b)}e^{\frac{\kappa}{V_T}V_1} - I_0^{(\mathrm{M}1)}e^{\frac{\kappa}{V_T}V_2} - I_0^{(\mathrm{M}1)}e^{\frac{\kappa}{V_T}V_3},\\
C\frac{\partial V_3}{\partial t} &= G_3\nabla^2 V_3 - I_3 + I_0^{(\mathrm{M}\alpha k)}e^{\frac{\kappa}{V_T}V_1} + I_0^{(\mathrm{M}\beta)}e^{\frac{\kappa}{V_T}V_2},
\end{aligned}$$

where V_i represents the node voltage, $I_{1,2,3}$ the injecting current, C the capacitance, κ the effectiveness of the gate potential, $V_T \equiv kT/q \approx 26$ mV at room temperature (k is Boltzmann's constant, T the temperature, and q the electron charge), and $I_0^{(\mathrm{M}i)}$ the fabrication parameter of nMOS FET. We fabricated analog LSI that electrically emulates the LV equation. The circuit took up a total area of 75 μm $\times$ 75 μm with a traditional 1.5-μm double-poly double-metal CMOS technology.
Results: We conducted circuit simulations of 50 $\times$ 50 2D diffusive LV circuits and observed its Turing-like spatiotemporal patterns. Initial values of each LV circuit were randomly chosen. After few iterations, all the values of each system variable became almost uniform. Then localized clusters appeared on the field. The clusters dilated the field, however, they were suddenly eroded towards their boundaries and created stiff clusters. Then the boundaries between the clusters were enhanced and the circuit system started to divide the enhanced clusters into multiple spots. The rest field was regularly filled with spots, as observed in well-known Turing systems. Generally the LV system takes very long time for its equilibrium. Therefore it's analog hardware implementation certainly helps us to explore and discover unknown physical properties on the diffusive LV systems; e.g., complex pattern formation, spot-pattern restoration, and so on.

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[1] Asai T. et. al., A MOS circuit for the LV chaotic oscillator, in *Proc. 12th Int. IEEE Workshop on Nonlinear Dynamics of Electronic Syst.,* 2004. (`http://133.87.128.168/int_conf/ndes_2004_asai.pdf`)

[2] Asai T., *et al.,* A subthreshold analog MOS circuit for LV chaotic oscillator, *Int. J. Bifur. Chaos,* in press.

[3] Mimura, M. & Kan-on, Y., "Predation-mediated coexistence and segregation structures," in *Patterns and Waves: Qualitative Analysis of Nonlinear Differential Equations*, pp. 129–155, 1986.