Chaotic Resonance in Forced Chua's Oscillator

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Stochastic resonance (SR) is one of the phenomena where dynamic noises are effectively used to induce state transition in a double-well potential system to which subthreshold driving forces are applied. The noises are given to the system as an additional force, which results in the requirement of extra noise sources.

Recently, a phenomenon called "chaotic resonance" (CR) has been spotlighted in the literature. CR can be observed in chaotic systems which have multiple strange attractors, and have ability to accept external forces to cause transitions between the attractors. In such chaotic systems, chaotic fluctuations, instead of external noises employed in SR systems, could effectively be used to cause the attractor transition, which may result in CR under certain parameter conditions; *i.e.*, CR systems may not require any external noise source, unlike traditional SR systems.

We employed Chua's oscillator as a possible candidate of CR systems because it has two attractors and when we adjust some system parameters, the attractors are combined into a single attractor, which is called double-scroll attractor. To induce CR, as in traditional SR system, we apply a sinusoidal driving force to the oscillator. The oscillator was implemented by discrete devices shown in Fig. 1, and the circuit's dynamics are described by

$$\begin{aligned} C_1 \frac{dv_{C_1}}{dt} &= G(v_{C_2} - v_{c_1}) - g(v_{C_1}) \\ C_2 \frac{dv_{C_2}}{dt} &= G(v_{C_1} - v_{c_2}) + i_L \\ L \frac{di_L}{dt} &= -v_{C_2} + A\sin(2\pi f t) \end{aligned}$$

where $A\sin(2\pi f t)$ represents the driving force, and $g(v_{C_1})$ the piecewise linear function given by

$$\begin{aligned} g(v_{C_1}) = m_0 v_{C_1} &+ \frac{1}{2}(m_1 - m_0)|v_{C_1} + B_P| \\ &+ \frac{1}{2}(m_0 - m_1)|v_{C_1} - B_P|. \end{aligned}$$

Negative resistor N_R was implemented by operational amplifiers based on [1]. The additional circuit parameters were set at $C_1 = 10$ nF, $C_2 = 100$ nF, $L = 20$ mH, $G = R^{-1} (R = 1.56\ \text{k}\Omega)$ and $A = 0.015$ (subthreshold input). Under these setups, we swept the frequency (f) of the driving force. When f was low, we could not observe attractor transition, as shown in Fig. 2 (a)-(b) (f = 40 Hz). Then, we increased the frequency, and when f = 140 Hz, we could observe the transition between two attractors (Fig. 2 (c)-(d)), where the output variable (x) was almost synchronized (and was resonated) with the input signal, which clearly exhibit chaotic resonance in the circuit. We will further investigate the relationship between the degree of the resonance (SNR, for example) and the power of generated chaotic noises.

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Reference

[1] Kennedy M. P., "Robust OP amp realization of Chua's circuit", FREQUENZ, vol. 46, no. 3-4, pp. 66-80, 1992.

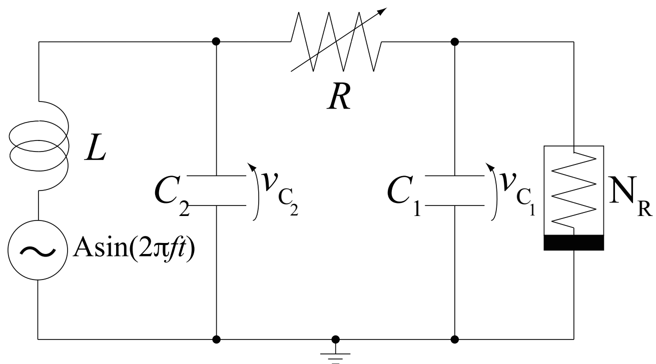

Fig. 1. Chua's circuit with external driving force.

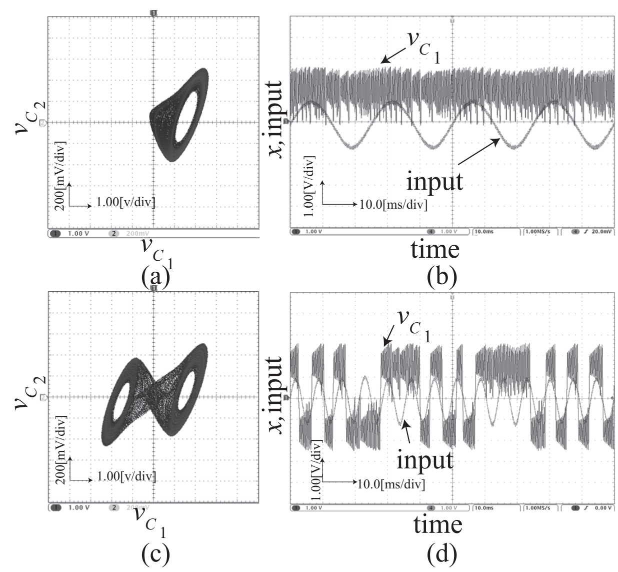

Fig. 2. Experimental results: phase-plane (a, c) and time-series (b, d)