NON-LINEAR DYNAMICAL SYSTEMS CONSISTING OF SINGLE-ELECTRON OSCILLATORS

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ABSTRACT- *The single electron circuit is as example of a non-linear dynamical system, with dual operation dynamics: continuous time and discrete time operation, resulting from coulomb blockade phenomenon, and electron tunneling. We designed a single electron oscillator, from which we constructed a double-oscillator system, and a quadruple-oscillator system. Through computer simulations, we confirmed that two oscillators coupled through a capacitor interacted with each other, generating synchronization and many-periodical oscillation but with a single attractor. We further confirmed that a quadruple-oscillator has a number of attractors which are initial-condition dependent. The dynamics of these systems are governed by a set of differential and discrete difference equations.*

1. INTRODUCTION

The single-electron circuit is an electronic circuit designed to manipulate electronic functions by controlling the transport of individual electrons, making use of the Coulomb blockade phenomenon [1]. Unlike ordinary electronic circuits, it changes its internal state discontinuously because of electron tunneling and consequently shows complex behavior expressed by a combination of continuous differential equations and discrete difference equations. In this paper, we take up systems consisting of coupled single-electron oscillators and illustrate their dynamics with the results of computer simulation. A single oscillator produces relaxation oscillation with a discontinuous jump in the variable value; a double-oscillator system produces many-periodical oscillation with a single attractor; and a quadruple-oscillator system has many possible attractors and takes one of them depending on its initial condition. The following sections provide the details on these results.

2. SINGLE-ELECTRON OSCILLATOR

The single-electron cell (Fig. 1(a)) is an elementary component of single-electron circuits. It consists of a tunneling junction C_j and a high resistance R connected in series at node 1 and biased with a positive voltage V_d. At low temperatures at which the Coulomb blockade is established (i.e., temperature << $e2/(2k_BC_j)$), the cell produces self-induced relaxation oscillation if $V_d > e/(2C_j)$, where e is the elementary charge and k_B is Boltzmann's constant. See [1, 2] for detailed explanation. Figure 1(b) shows the waveform of the oscillation in voltage V_1 at node 1. The node voltage gradually increases as junction capacitance C_j is charged through resistance R (curve AB). When the voltage reaches the threshold $e/(2C_j)$, it drops discontinuously to $-e/(2C_j)$ because of an electron tunneling from the ground to node 1 through the junction, again gradually increasing to repeat the same cycles.

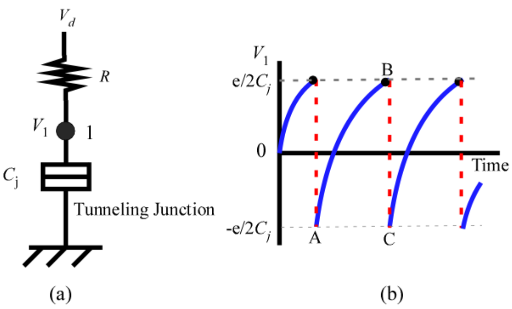

Fig. 1. Single electron oscillator:
(a) structure and (b) oscillation waveform

The dynamics of the single-electron oscillator are expressed by a combination of continuous differential equation

$$\frac{dV_1}{dt} = \frac{V_d - V_1}{RC_j} \qquad (1)$$

for charging curve AB and discrete difference equation

$$\Delta V_1 = \frac{e}{C_j} \qquad (2)$$

for discontinuous drop BC, where ΔV_1 is the difference in the node voltage before and after tunneling.

3. DOUBLE-OSCILLATOR SYSTEM
---- a combination of two oscillators

Coupling positively biased and negatively biased oscillators produces interaction between the oscillators and consequently causes complex dynamics of the system. Figure 2 shows the double-oscillator system we proposed previously [3]. The system consists of a positively biased oscillator (left) and a negatively biased oscillator (right) coupled with a capacitance C. The two oscillators interact with each other through the coupling capacitance: for example, if electron tunneling occurs in the left oscillator from the ground to node 1, then node 1 carries a negative charge to decrease its voltage to a negative, and this induces tunneling in the right oscillator from node 2 to the ground. The variables of this system are node voltages V_1 and V_2.

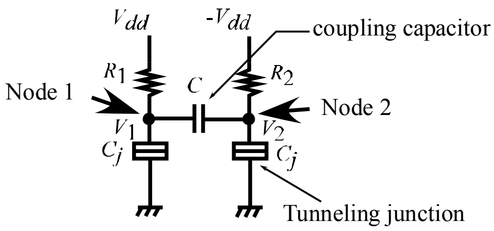

Fig. 2. Double oscillator system consisting of two coupled oscillators.

To express the dynamics of the system, we rewrite the variables and parameters as follows:

$u_i = 2C_0V_i/e$ (i = 1,2) , $C_0 = (2k+1)Cj/(k+1)$,
$k = C/Cj$, $\alpha = R_2/R_1$, $\beta = 2C_0V_{dd}/e$, and
t = time $/R_1C_0$,

where u_1, and u_2 are normalized node voltages, k is a coupling coefficient ($k \geqq 0$), and t is normalized time.

With this rewriting, the dynamics of the system are given by differential equations

$$\frac{du_1}{dt} = (\beta - u_1) - \frac{k}{k+1}\frac{1}{\alpha}(\beta + u_2) \quad and$$

$$\frac{du_2}{dt} = \frac{k}{k+1}(\beta - u_1) - \frac{1}{\alpha}(\beta + u_2) \qquad (3)$$

for $-1 < u_1 < 1$ and $-1 < u_2 < 1$ and difference equations

$$\Delta u_1 = -2, \quad and \quad u_2 = -\frac{2k}{2k+1} \quad if \quad u_1 \quad reaches \quad 1, \qquad (4)$$

$$\Delta u_2 = -2, \quad and \quad u_1 = -\frac{2k}{2k+1} \quad if \quad u_2 \quad reaches \quad 1, \qquad (5)$$

$$\Delta u_1 = 2, \quad and \quad u_2 = \frac{2k}{2k+1} \quad if \quad u_1 \quad reaches \quad -1, \qquad (6)$$

$$\Delta u_2 = 2, \quad and \quad u_1 = \frac{2k}{2k+1} \quad if \quad u_2 \quad reaches \quad -1. \qquad (7)$$

The double-oscillator system produces many-periodical oscillation with a single attractor on a u_1- u_2 phase plane. The attractor consists on a number of segments, depending on parameter values. Figure 3 gives an example, showing a change in the number of the segments with coupling coefficient k as the parameter. To show a general image of the effect of coupling strength, we give a sample bifurcation diagram in Fig. 4, where the values of u_2 at which the attractor meets line $u_1 = 1$ are plotted as a function of k. The degree of periodicity increased as k increases, but windows appeared repeatedly.

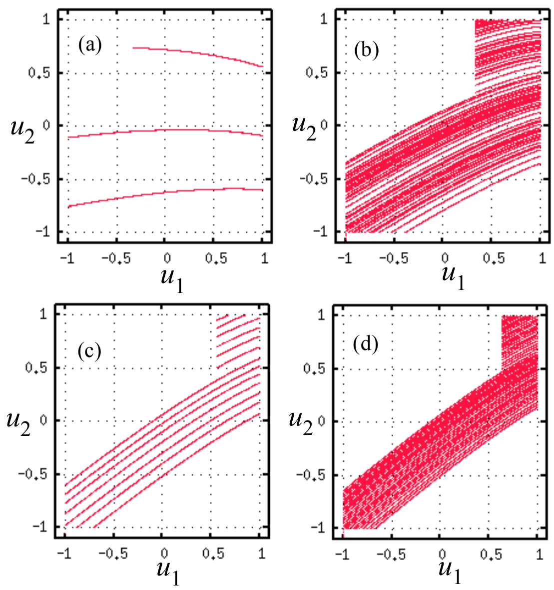

Fig. 3. Phase portraits showing attractors of the oscillation, simulated for (a) $k = 0.5$, (b) $k = 2.0$, (c) $k = 3.6$, and (d) $k= 4.6$, with $\alpha = 10^{0.5}$, and $\beta = 3.0$.

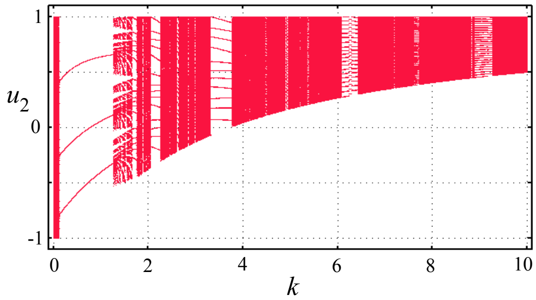

Fig. 4 . Bifurcation diagram for $\alpha = 10^{0.5}$ and β=3.0 , simulated with coupling coefficient k as the bifurcation parameter.

4. QUADRUPLE-OSCILLATOR SYSTEM
----a combination of two double-oscillator systems

Coupling two double-oscillator systems will produce a new system with more complex dynamics. On the basis of this idea, we propose a single-electron system shown in Fig. 5. The system consists of four oscillators connected in a ring with four coupling capacitances C. The variables of this system are four node voltages V_1, V_2, V_3, and V_4.

To express the dynamics of the system, we rewrite the variables and parameters as

$v_i = 2C_s V_i/e$ (i = 1- 4) , $C_s = (8k^2 + 6k + 1)C_j/(2k^2 + 4k + 1)$,

$k = C/C_j$, $\alpha = R_2/R_1$, $\beta = 2C_s V_{dd}/e$, and

t = time $/R_1C_s$, where v_i is the normalized node voltage, k is the coupling coefficient ($k \geqq 0$), and t is normalized time.

With this rewriting, differential equations for the system are given by

$$\frac{dv_1}{dt} = -\frac{2(\gamma+\zeta-\kappa-\eta)k^2 + (-4\kappa+\gamma+\zeta)k - \kappa}{(4k+1)(2k+1)}$$

$$\frac{dv_2}{dt} = -\frac{2(\gamma+\zeta-\kappa-\eta)k^2 + (4\gamma-\kappa-\eta)k + \gamma}{(4k+1)(2k+1)}$$

$$\frac{dv_3}{dt} = -\frac{2(\gamma+\zeta-\kappa-\eta)k^2 + (-4\eta+\zeta+\gamma)k - \eta}{(4k+1)(2k+1)}$$

$$\frac{dv_4}{dt} = -\frac{2(\gamma+\zeta-\kappa-\eta)k^2 + (4\zeta-\kappa-\eta)k + \zeta}{(4k+1)(2k+1)} \quad (8)$$

for $-1 < v_i < 1$, where

$$\gamma = \frac{\beta + v_2}{\alpha} \qquad \zeta = \frac{\beta + v_4}{\alpha}$$

$$\kappa = \beta - v_1 \qquad \eta = \beta - v_3 \quad (9)$$

When either of the node voltages v_i reaches ± 1, electron tunneling occurs between the corresponding node and the ground, and this changes the voltage of the node and the other three nodes. For example, if tunneling occurs between node 1 and the ground, the discontinuous change in v_i is given by

$$\Delta v_1 = -2 \quad (10)$$

for node 1,

$$\Delta v_{adj} = -\frac{2k(2k+1)}{2k^2 + 4k + 1} \quad (11)$$

for the adjacent neighboring nodes 2 and 4, and

$$\Delta v_{diag} = -\frac{4k^2}{2k^2 + 4k + 1} \quad (12)$$

for the diagonally positioned node 3, when v_1 reached +1. Otherwise, if v_1 reached -1, the corresponding node voltages would change by a positive value of the voltages given in Eq. 8 – 10 above.

Tunneling at the other nodes would also lead to a similar change of node voltages.

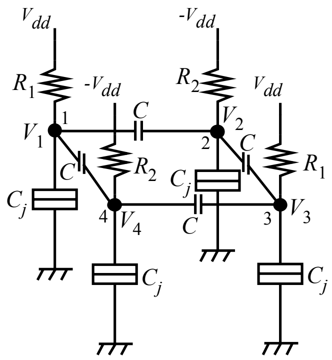

Fig. 5. Quadruple-oscillator system consisting of four coupled oscillators.

We simulated the operation of the system for various sets of parameters and found that this system showed many-periodical oscillation with a number of possible attractors instead of a single attractor. The attractor the system actually takes was determined by the initial values of the four node voltages v_1 through v_4: in other words, the attracter of the system was initial-condition dependent. An example is illustrated with Fig. 6(a), a diagram showing the dependence of the degree of oscillation periodicity on the initial value of node voltages. This diagram was obtained as follows. First, we calculated the attractor of the oscillation in a four-dimensional v_1-v_2-v_3-v_4 phase space for various sets of initial node voltages. Then we projected the attractor onto the v_1 - v_2 plane to reduce its dimensions. The projected attractor consisted of a number of segments. Figures 6(b-1) and 6(b-2) show the projected attractor for two sets of initial node voltages. After that, we counted the number of the segments composing the attractor. It depended on the initial condition; some sets of initial condition resulted in few number of segments, while others resulted in many segments. Finally, we plotted the number of the segments on an initial-condition plane. For simplicity, we assumed that the initial voltages of nodes 1 and 3 (and initial voltages of nodes 2 and 4) were the same in amplitude and inverse in polarity; therefore the initial-condition plane could be reduced to a v_1-v_2 plane. The resultant diagram is shown in Fig. 6(a), representing the number of segments by gray scale: the lightly shaded region means large number of segments, and the dark one means few segments. Another set of parameters produced a different diagram as shown in Fig. 7.

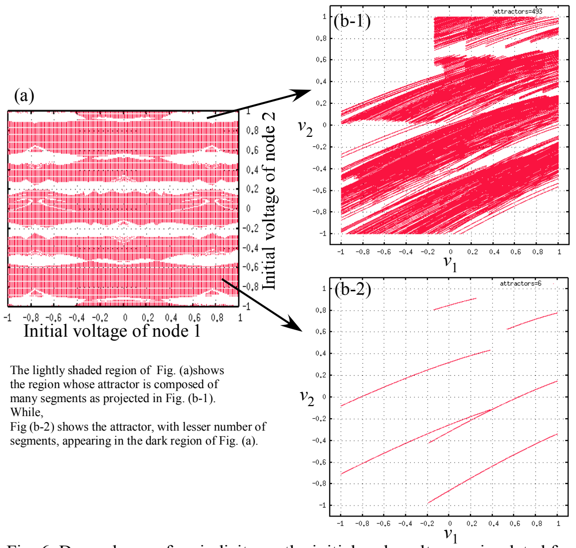

Fig. 6. Dependence of periodicity on the initial node voltages, simulated for $k = 1.0$, $\alpha = 10^{0.5}$, and $\beta = 3.0$.

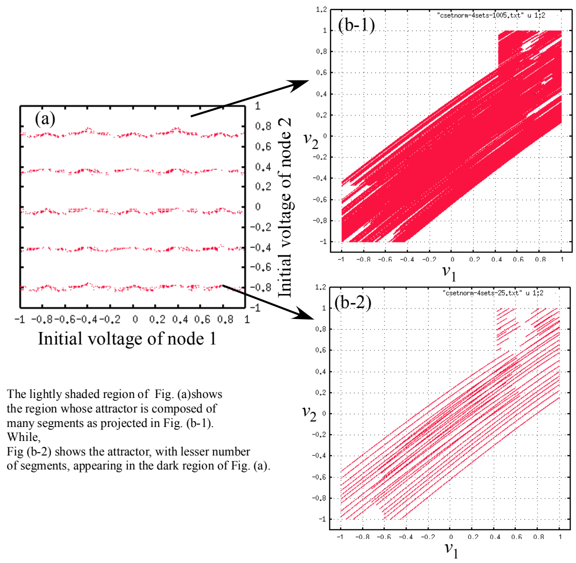

Fig. 7. Dependence of periodicity on the initial node voltages, simulated for $k = 3.6$, $\alpha = 10^{0.5}$, and $\beta = 3.0$.

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