An analog CMOS circuit implementing Turing's reaction-diffusion model

Takahiro DAIKOKU, Tetsuya ASAI, and Yoshihito AMEMIYA

Department of Electrical Engineering, Hokkaido University
Kita 13, Nishi 8, Sapporo, 060-8628 Japan
daikoku@sapiens-ei.eng.hokudai.ac.jp

Abstract *−* We propose an analog CMOS LSI that implements the pattern formation in Turing's reaction-diffusion systems. The circuit consists of *reaction* circuits that imitate chemical reactions and *diffusion* circuits that imitate the diffusion of chemicals. It acts as a spatial-frequency filtering device and can be used in signal processing such as the restoration and enhancement of input images.

(Keyword: Turing model, LSI, MOS, image processing)

I. Introduction

Turing system is a reaction-diffusion system that generates stationary, spatially periodic patterns called Turing patterns. It consists of two chemicals that control the synthesis rate of each other. If the chemicals react and diffuse in an appropriate way, spatial patterns of their concentrations can arise from an initial distribution with a small disturbance. This phenomenon can be applied to image preprocessing such as enhancement and restoration of texture patterns [1-3]. An example is shown in Fig. 1.

To develop novel image processing devices, we propose constructing a CMOS LSI that imitates the dynamics of Turing systems. The LSI, a Turing chip, consists of regularly arrayed cell circuits, i.e., reaction cells and diffusion cells: the reaction cell imitates local reaction, and the diffusion cell imitates diffusion between adjacent reaction cells.

We designed the cell circuits that make use of the transfer characteristic of MOS-transistor differential pairs. In these circuits, the concentrations of two chemicals are represented by two voltage signals. As a first step toward the LSI, we combined the reaction cells and diffusion cells into a ring to construct a circuit that imitated a one-dimensional Turing system. Computer simulation showed that, given a blurred one-dimensional pattern as an initial input, the circuit successfully produced a restored, enhanced pattern.

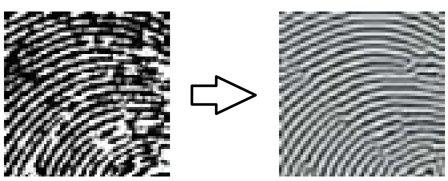

Fig. 1 Restoring a noisy fingerprint pattern on the basis of Tusing's model.

II. Implementing Turing systems on a LSI chip

The spatial and temporal dynamics of reaction-diffusion systems are given by

$$\frac{\partial u_i}{\partial t} = f_i(u_1, u_2, \ldots, u_n) + D_i \nabla^2 u_i \quad (i = 1, 2, \ldots, n) \quad \ldots.(1)$$

where variable u_i and constant D_i are the concentration and diffusion coefficient of the i-th chemical substance. The function f_i (u_1, u_2, …) in the right-hand side is the reaction term that represents the effect of chemical reaction and the spatial derivative is the *diffusion* term that represents the diffusion of the substance .

The simplest example is a two-variable system with a linear reaction function (a linear function suffices for the pattern formation in Turing systems). The equation for this simple system is

$$\begin{cases} \dfrac{\partial u}{\partial t} = au - bv + D_u \nabla^2 u \\ \dfrac{\partial v}{\partial t} = cu - dv + D_v \nabla^2 v \end{cases} \quad \ldots\ldots (2)$$

where u and v are the concentrations of the two substances, and D_u and D_v are the diffusion coefficients of the substances. The parameters a, b, c, and d in the reaction terms determine the dynamics of the system. If all reaction parameters are set positive (a, b, c, $d > 0$), variable u acts as an activator and variable v an inhibitor. Under appropriate conditions (i.e., $a < d$, $bc > ad$, and $D_v >> D_u$), the system produces Turing patterns.

To construct the Turing system on an LSI chip, we represent variables u and v by voltage signals and imitate chemical reaction and diffusion by CMOS circuits. The schematic image of the LSI we propose is illustrated in Fig. 2. The LSI consists of reaction cell circuits that imitate reaction and diffusion cell circuits that imitate diffusion. Both cell circuits are regularly arrayed on a chip and interconnected by wires that carry the voltage signals; each reaction cell is connected with its neighboring reaction cells through the diffusion cells.

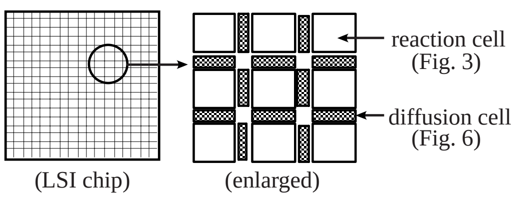

Fig. 2 Turing LSI consisting of reaction cells and diffusion cells.

III. Reaction cell circuit

We designed the cell circuits to implement the Turing dynamics expressed by equation (2). We made use of the transfer characteristic of MOS-transistor differential pairs.

Figures 3 shows the reaction cell circuit. It consists of four differential pairs and four integral capacitors C. The two variables are represented by differential voltages u and v on the signal line. The four differential pairs correspond to the four reaction terms in equation (2); i.e., the leftmost differential pair corresponds to reaction term au; the second to $-bv$, the third to cu, and the rightmost to $-dv$.

The transfer characteristic of a differential pair (Fig. 4) is given by

$\Delta I = F(u)$ (ΔI: output current, u: input voltage)

$$F(u) = k_1\, u\,(1 - u^2/(2\,k_2^2))^{1/2} \quad \text{for} \quad -k_2 < u < k_2$$
$$F(u) = k_1\, k_2 /2^{1/2} \quad \text{for} \quad u > k_2 \qquad .....(3)$$
$$F(u) = -k_1\, k_2 /2^{1/2} \quad \text{for} \quad u < -k_2\,,$$

where positive coefficients k_1 and k_2 are a function of MOS-transistor gain factor β and bias current I_0 and given by $k_1 = (\beta I_0)^{1/2}$ and $k_2 = (2I_0/\beta)^{1/2}$. The dynamics of the reaction cell circuit can therefore be expressed as

$$\begin{cases} C\dfrac{du}{dt} = F_1(u) - F_2(v) \\ C\dfrac{dv}{dt} = F_3(u) - F_4(v) \end{cases} \qquad .....(4)$$

where $F_1(u)$ through $F_4(v)$ are the transfer functions (corresponding to $F(u)$ in equation (3)) of the four differential pairs. The transfer functions are linear when the value of voltages u and v are small (see Fig. 4), so the reaction terms in equation (2) can be implemented. The reaction parameters a, b, c, and d can be controlled by adjusting the MOS-transistor gain factor and the bias current in each differential pair.

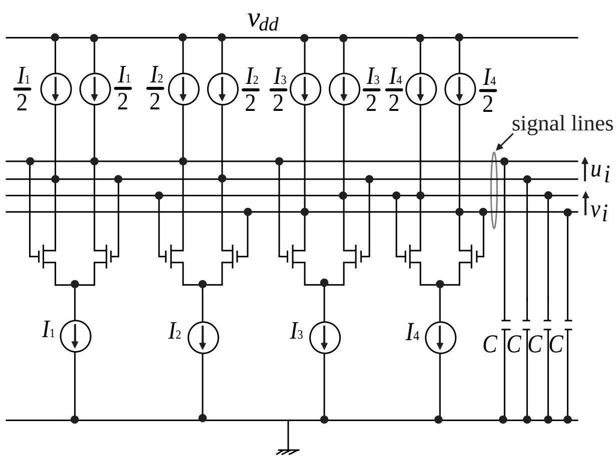

Fig. 3 The reaction cell circuit consisting of four differential pairs and four integral capacitors.

We confirmed the operation of the cell circuit by computer simulation. We used circuit simulator HSPICE and 1.2-μm CMOS device parameters. Depending on the reaction parameters, the circuit can be oscillatory or stationary. Three examples of the cell operation are shown in Figs. 5(a)-5(c) on the u-v phase plane, with the values of the reaction parameters. The bias current was set to 50 μA for all four differential pairs. The capacitance of the integral capacitor was set to 10 pF. We controlled the reaction parameters by adjusting the MOS-transistor gate width in each differential pair. For generating Turing patterns, we use the circuit under stationary conditions.

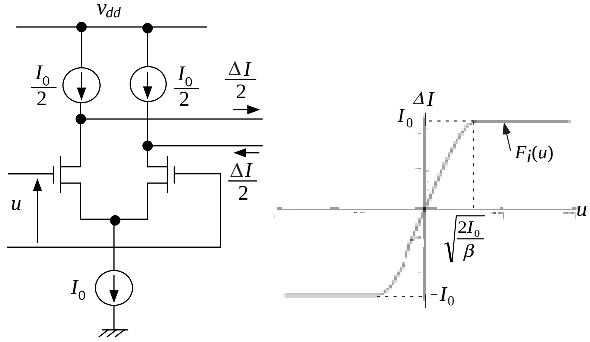

Fig. 4 The transfer characteristic of a differential pair.

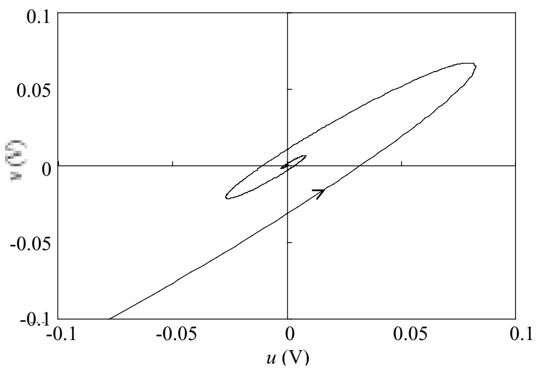

Fig. 5(a) Trajectory converging to the origin on the u-v phase plane (the ratio between reaction parameters a:b:c:d =1:1.2:1: 1.1).

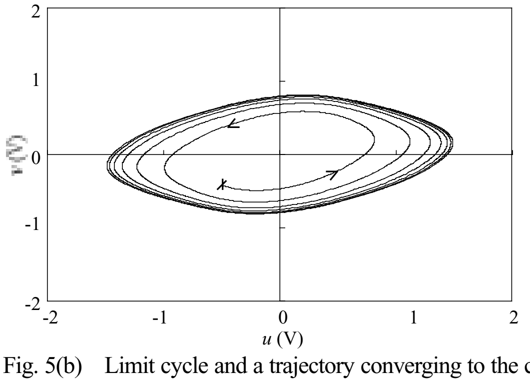

Fig. 5(b) Limit cycle and a trajectory converging to the cycle on the u-v phase plane (the ratio between reaction parameters a:b:c:d =1:2.3:1:0.7).

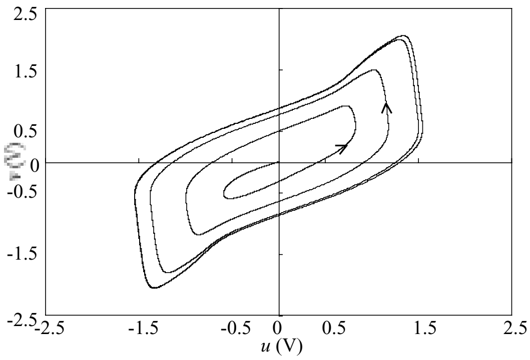

Fig. 5(c) Limit cycle and a trajectory converging to the cycle on the u-v phase plane (the ratio between reaction parameters a:b:c:d =1:1:1:0.7).

IV. Diffusion cell circuit

Figure 6 shows the diffusion cell circuit consisting of four differential pairs. The circuit has two pairs of differential signal lines to couple with two adjacent reaction cells; the left pair (u_i and v_i) are connected to the signal lines of one reaction cell, and the right pair (u_{i+1} and v_{i+1}) are connected to the other reaction cell. The circuit operates as a pair of *floating resistors* and conducts differential currents (ΔI_u and ΔI_v) from one side to the other as shown in the figure. The currents are proportional to the difference between the signal voltages (u_i–u_{i+1} and v_i–v_{i+1}) when the voltage signals are small, so we can imitate diffusion with this cell circuit. We can control the diffusion coefficient by adjusting the MOS-transistor gain factor and the bias current in each differential pair.

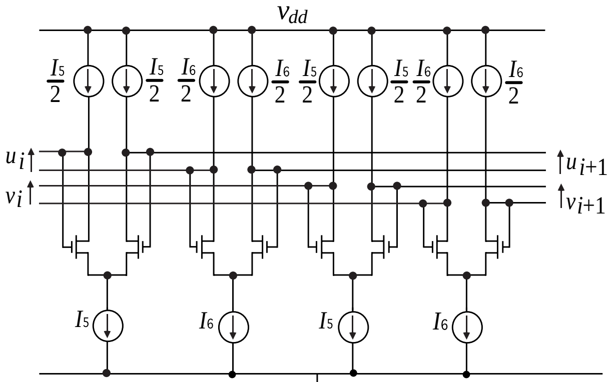

Fig. 6 The diffusion circuit consisting of four differential pairs.

To confirm the operation of the circuit, we combined many diffusion cells into a chain and simulated the "diffusion" of a voltage signal through the chain. One of the results is shown in Fig. 7. In this example, we connected 50 diffusion cells and also connected a 10-pF capacitor to each connection point of adjacent cells. Then, at time = 0, we applied a step voltage of 0.6 V to the first cell of the chain (i.e., $u_1 = v_1 = 0.6$ V at time =0). The figure shows the distribution of the signal

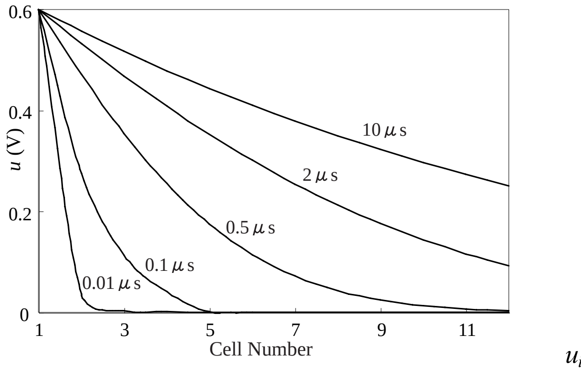

Fig. 7 Diffusion of a voltage signal through a chain of the diffusion cells. Step voltage of 0.6 V was applied to the first cell, or cell 1

voltage u_i through the chain, with time as a parameter. The voltage signal is transported by "diffusion" through the chain of the cells.

V. Constructing Turing systems

As a first step toward Turing LSIs, we combined the reaction cells and diffusion cells into a ring to construct the circuit that imitates a one-dimensional Turing system. The configuration of the ring circuit is illustrated in Fig. 8.

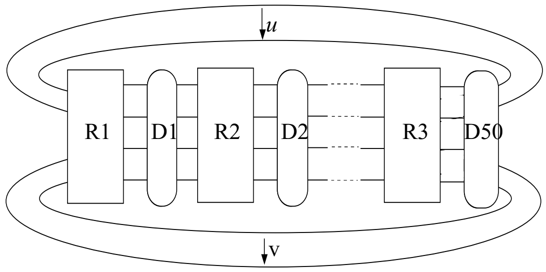

Fig. 8 One-dimensional Turing system consisting of the reaction cells and the diffusion cells connected into a ring (R1-R50 : reaction cells, D1-D50 : diffusion cells).

Computer simulation confirmed the generation of Turing patterns in this circuit. Figures 9(a)-9(c) illustrate the results for the circuit with 50 reaction cells and 50 diffusion cells. We gave initial voltage signals u_i and v_i to each reaction cell as shown in Fig. 9(a), where u_i in each reaction cell is plotted. Then we left the circuit changing its state without restraint. After some transition time, the circuit stabilized in a final state as shown in Fig. 9(b). The pattern that is formed by cell voltages u_i is a one-dimensional Turing pattern. The spatial frequency of the pattern can be controlled by the parameters (reaction parameters and diffusion coefficients). Figure 9(c) shows a Turing pattern with another set of parameters.

The circuit acts as a spatial-frequency filtering device and can be used in signal processing such as the restoration and enhancement of one-dimensional texture images. The simulated result is shown in Figs. 10(a) and 10(b). The upper stripe in Fig. 10(a) is a given blurred picture. To restore this picture, we converted the shade pattern of the picture into a voltage pattern u_i as depicted in Fig. 10(b) by a dashed curve. Then we put the voltage u_i in each reaction cell (voltages v_i were set to 0) and left the circuit changing its state without restraint. After some transition time, voltages u_i in the cells stabilized in their final values to form a pattern as shown by a solid curve in Fig. 10(b). By reconverting the voltage pattern to a shade pattern, we obtained restored, enhanced picture as shown by the bottom stripe in Fig. 10(a). Our results will open prospects for developing novel image-processing LSIs.

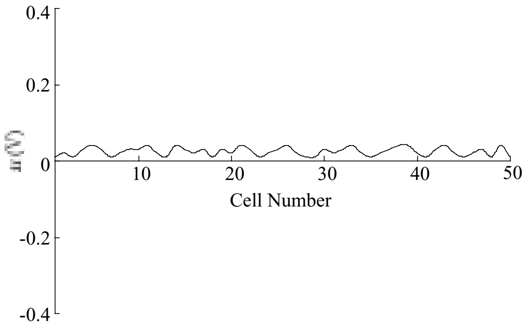

Fig. 9(a) Initial distribution of voltage signal u_i in the ring circuit. Cell numbers 0 and 50 denote the identical cell.

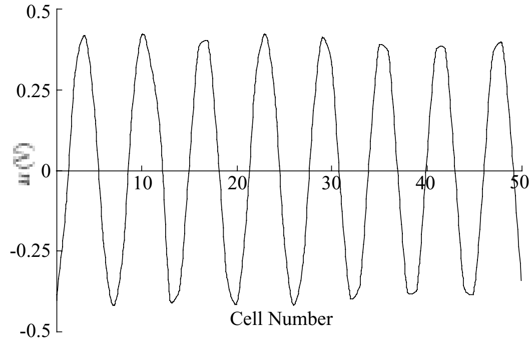

Fig 9(b) Turing pattern in the ring circuit (the ratio between reaction parameters a:b:c:d =1:2:1:1.5, and the ration between diffusion coefficients Du : Dv = 1: 10).

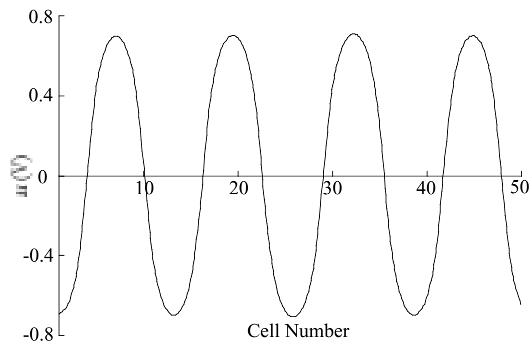

Fig 9(c) Turing pattern in the ring circuit with another set of parameters (a:b:c:d =1:1.2:1:1.1, and Du : Dv = 1: 10).

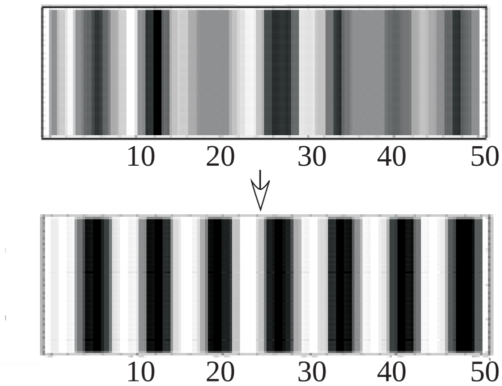

Fig. 10(a) Restoration and enhancement of a given one dimensional pattern. The upper stripe is an initial blurred picture, and the bottom stripe is the restored, enhanced picture. (The parameters of the circuit are a:b:c:d =1:1.2:1:1.1 and Du : Dv = 1:3).

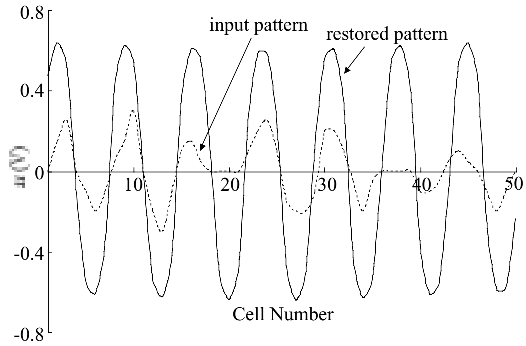

Fig. 10(b) The pattern of cell voltages u_i over 50 cells from cell 1 to cell 50. The dashed line corresponds to the blurred input picture in Fig. 10(a), and the solid line corresponds to the restored, enhanced picture in Fig. 10(a).

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