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Digital VLSI Implementation of Ultra-Discrete Cellular Automata for Simulating Traffic Flow

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Abstract— **We propose digital VLSI implementation of multiple-value cellular automata for simulating traffic flow. Recently, a family of the Burgers cellular automata (BCA) has been proposed as multi-lane traffic flow models, which are derived from the Burgers' equation interpreted as a macroscopic traffic flow model using the ultra-discrete method. The family of BCA is suitable for digital VLSI implementation because of the discreteness and the simple update procedure. For developing efficient traffic simulators, we implemented the family of BCA as digital VLSI circuits using a scalable CMOS technology. Using computer simulator SPICE, it is shown that these circuits operate correctly, and they can be expected to be useful tools to analyze and predict the behavior of traffic flow**

I. INTRODUCTION

Traffic flow phenomena have recently attracted much attention from both engineers and physicists [1]. It shows a wide variety of collective behaviors such as pattern formation, phase transition and scale invariant fluctuations [2]-[3]. Especially, dynamic phase transition between free to congested traffic flow, such as traffic jam, is the most significant problem from both the applied and theoretical points of view.

In order to investigate traffic flow phenomena in detail, numerous traffic models have been proposed: car-following models [4]-[5], coupled-map lattice models [6], particle-hopping models [7]-[10], gas-kinetic models [11]-[12], and fluid-dynamic models [13]-[14]. Such models can be classified into three categories: microscopic, macroscopic, and mesoscopic ones. Microscopic models are based on the interactions between vehicles responsible for traffic flow. Macroscopic models are based on the average movement of vehicles characterized by the fundamental relationships between vehicle speed, flow, and density. Mesoscopic models are intermediated.

Cellular automata (CA) have been used as microscopic particle-hopping models [7]-[10]. A cellular automaton is a quite simple dynamical system that consists of elements, called cells, in a finite number of states in a discrete space and discrete time. The states of cells are updated according to the states of neighboring cells in synchrony. Despite the simple update procedure, CA shows a wide variety of collective behaviors and can simulate complex physical processes. The most fundamental CA model for traffic flow is the rule-184 CA model, which is one of the elementary CA classified by Wolfram [16]. In the model, a road is divided into cells that can be empty or occupied by a car, and each car moves forward on each time step. Several extended versions of the rule-184 CA have been proposed [7]-[10]. For instance, Nagel and Schreckenberg proposed a stochastic CA for modeling single-lane traffic flow [7]. Fukui and Ishibashi introduced a deterministic CA model [8]. These models show a phase transition between free to congested traffic flow. However, it is difficult to extend these models to multi-lane traffic flow models.

Recently, a class of multiple-value CA, based on the Burgers CA (BCA) has been proposed as multi-lane traffic flow models [10]. BCA is derived from the Burgers' equation [15] interpreted as a macroscopic traffic flow model by using a new type of discrete method, so-called the ultra-discrete method [10]. Thus, BCA inherits macroscopic properties of the original equation. It also includes an extended version of the rule-184 CA in a special case.

Due to the discreteness and the simple update procedure, the family of BCA is efficient in computing and hardware implementation. Especially, the family of BCA is suitable for digital VLSI implementation in terms of the following points: (i) All variables are small positive integer, (ii) their update procedures are expressed as explicit difference equations by using 'add' and 'min' functions, and (iii) each cell has same terms with neighboring ones and this fact supports parallel circuit architecture.

Focusing on such advantages, we implemented two versions of BCA as digital VLSI circuits toward developing efficient traffic flow simulators. We designed these circuits using a scalable CMOS technology. Through computer simulations, we show that the circuits operate correctly, and they can be expected to be useful tools to analyze and predict the collective behaviors of traffic flow

II. BURGERS CA MODELS FOR TRAFFIC FLOW

The family of Burgers CA (BCA) [10] has been derived from the Burgers' equation [15]:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} \tag{1}$$

which is used for modeling viscous fluid flow as well as traffic flow [14]. By applying the ultra-discrete method to the

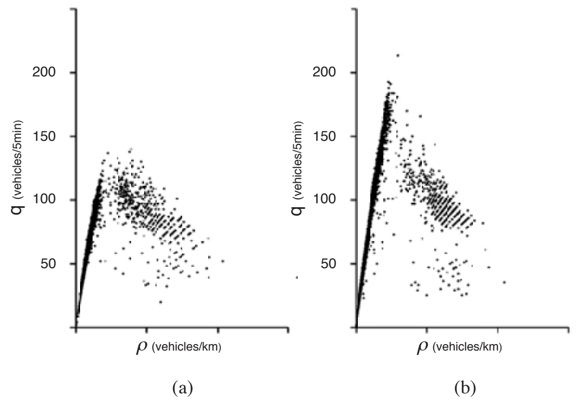

Fig. 1. Fundamental diagrams of real traffic flow on the Tomei expressway. (a) Driving lane and (b) acceleration lane cited from [10].

Burgers' equation, we can obtain BCA given by the following equation [10]:

$$U_j^{t+1} = U_j^t + \min(M, L - U_{j-1}^t, U_j^t) - \min(M, L - U_j^t, U_{j+1}^t) \quad (2)$$

where U_j^t represents the state of a cell j at a time t , L and M the parameters, and min ( ) the minimum function. If it is assumed that M $\geq$ 0, L $\geq$ 0 and 0 $\leq U_j^t \leq$ L for any cell j at a certain time t, then 0 $\leq U_j^t \leq$ L holds for any j. In this case, (2) can be regarded as a multiple-value CA. If we put a restriction L $\leq$ M on (2), it can be rewritten as follows:

$$U_j^{t+1} = U_j^t + \min(L - U_{j-1}^t, U_j^t) - \min(L - U_j^t, U_{j+1}^t) \quad (3)$$

where it is equivalent to the rule-184 CA, which is extended to several traffic CA models, such that L =1. It can be also regarded as an extended version of the rule-184 CA with a multiple value set {0,…,L}. Thus, it is natural to consider (2) as a microscopic model for multi-lane traffic flow. In the case, the road is divided into cells that can be empty or occupied by cars, and each car moves forward on each time step. U_j^t corresponds the number of cars, L is interpreted as the capacity of cars at each cell, i.e., the number of lanes, and M is the maximum number of movable cars [10].

As a traffic flow model, it is desirable to show same behaviors observed in real traffic, such as pattern formation and phase transition between free to congested flow. Traffic flow is characterized by the traffic flow variables: vehicle speed v, density ρ, and flow q. Traffic flow is investigated in terms of relationships between these variables in detail. For instance, there exists the following relationship: $q = \rho v$. The fundamental diagram that describes these relationships is often used to analyze traffic flow. Figure 1 shows typical fundamental diagrams, flow-density relationships, obtained from empirical data of highway traffic [10]. We can find phase transition from free flow at low density to congested flow at high density.

The fundamental diagram obtained from calculation data of the BCA model is as shown in Fig. 2(a). The flow and the density are defined as follows [10]:

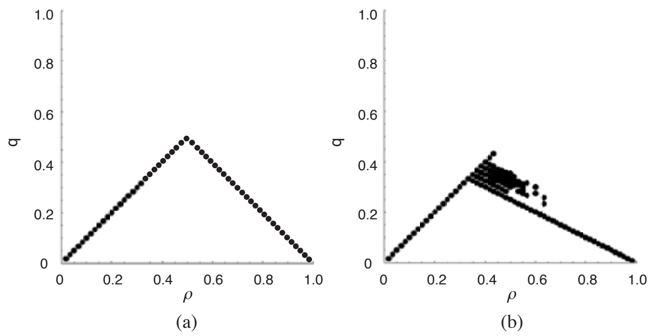

Fig. 2. Fundamental diagrams of two variants of BCA. (a) BCA and (b) multiple-value SlS CA with L=2 and K = 30 [10].

$$\rho^t \equiv \frac{1}{KL}\sum_{j=1}^{K} U_j^t \, , \quad q^t \equiv \frac{1}{KL}\sum_{j=1}^{K} \min(M, U_j^t, L - U_{j+1}^t) \quad (4)$$

where K represents the total number of cells. Figure 2(a) indicates that BCA has a critical point that occurs a simple phase transition.

Recently, an extended version of BCA for traffic flow model that introduces slow-to-start effects has been proposed. Its evolution dynamics is given by the following equations [10].

$$U_j^{t+1} = U_j^t + S_j^t - S_{j+1}^t \quad (5)$$

$$S_j^t = \min[U_{j-1}^t - \{U_{j-1}^{t-1} - \min(L - U_j^{t-1}, U_{j+1}^{t-1})\}, L - U_j^{t-1}] \quad (6)$$

where if L = 1, it becomes equivalent to the slow-to-start (SlS) CA model [9], and thus it can be regarded as a multiple-value SlS CA model [10]. Figure 2(b) shows the fundamental diagram obtained from calculation data of the multiple-value SlS model, where the flow and the density is defined as follows [10]:

$$q^t \equiv \frac{1}{KL}\sum_{j=1}^{K} S_j^t \, . \quad (7)$$

As shown in Fig. 2(b), the multiple-value SlS CA model show complex phase transitions as observed in empirical data.

III. Circuit Architecture

We implemented two variants of BCA as digital VLSI circuits. Due to their discreteness and simple update procedure, they are efficient in computing and hardware implementation. Especially, these CA models are suitable for d

igital VLSI implementation in terms of the following points: (i) All variables are small positive integer, (ii) their update procedures are expressed as explicit difference equations by using 'add' and 'min' function, and (iii) each cell has same terms with neighboring ones and this fact supports parallel circuit architecture.

First, we constructed a BCA circuit with adders, minimum circuits, and flip-flops. Figure 3 shows the block diagram of the BCA circuit. The 2-bit minimum circuit, shown in Fig. 4,

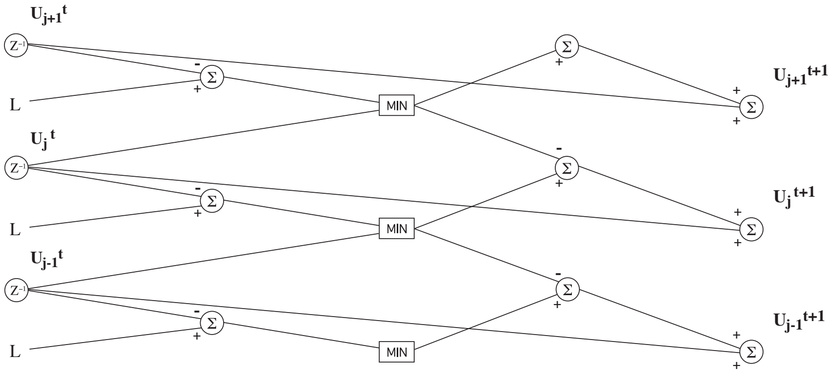

Fig. 3. Block diagram of BCA circuit.

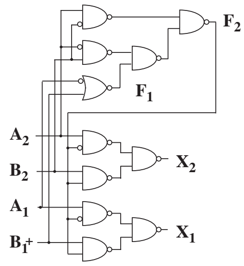

Fig. 4. Minimum circuit.

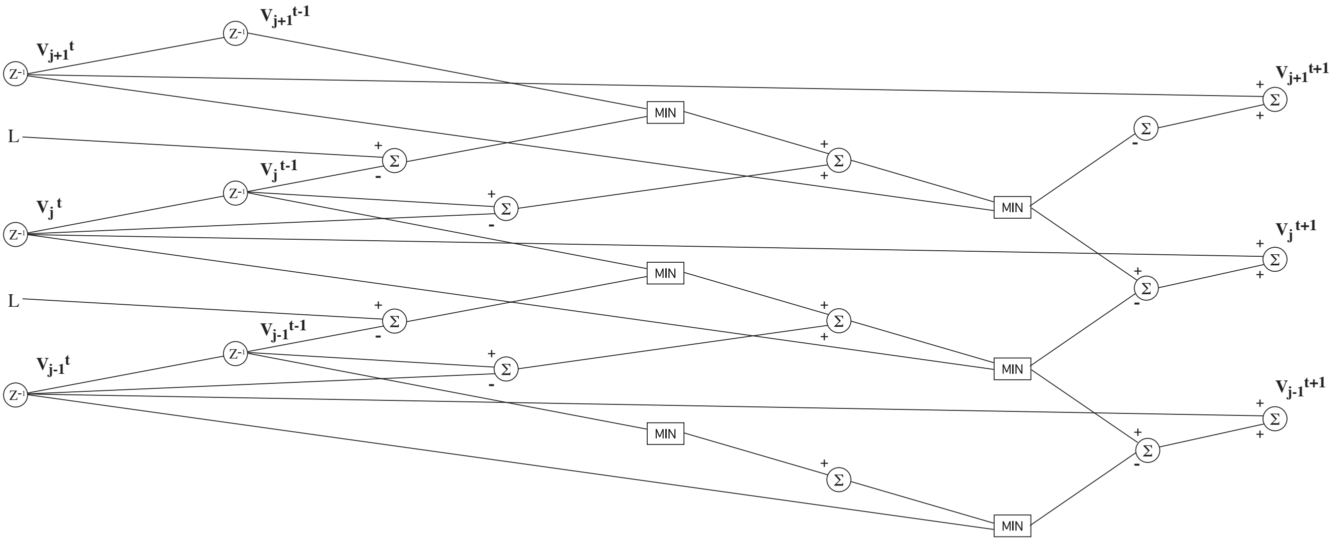

Fig. 5. Block diagram of multiple-value SlS CA circuit.

consists of a selector and a comparator that outputs:

$$F_2 = A_2 \cdot \bar{B}_2 \vee (A_2 \vee \bar{B}_2)F_1,\;\; F_1 = A_1 \cdot \bar{B}_1 \qquad (8)$$

where F_2 =1 if A = (A_2, A_1) > B = (B_2, B_1). Depending on the output of the comparator F_2, the selector outputs min (A, B). In practical, the minimum circuit consists of NOT, NAND, and NOR gates. If we consider BCA as a highway traffic model, all variables are smaller integer than L $\leq$ 3 and these can be represented as at most 2-bit in binary. As for adders, it is need at most 3-bit to represent variables, taking into account LSB carry and 2' complementary in binary for subtraction.

Then, we constructed a multiple-value SlS CA circuit as shown in Fig. 5. For constructing the circuit, we applied the lS CA model to the following transformation:

$$V_j^t = L - U_j^t \qquad (9)$$

and then the evolution equations are rewritten as follows:

$$V_j^{t+1} = V_j^t + S_{j+1}{}^t - S_j{}^t \qquad (10)$$

$$S_j{}^t = \min[V_{j-1}^{t-1} - V_{j-1}^{t+1} + \min(L - V_j^{t-1}, V_{j-1}^{t-1}), V_j^{t-1}]. \qquad (11)$$

As a result of this simple transformation, one stage of adder circuits can be reduced. If we would read outputs from the circuit, translation of the outputs should be required.

IV. Simulation and Results

We confirmed the operation of the proposed circuits using the circuit simulator SPICE. In the following simulations, we assumed a standard scalable CMOS process (TSMC 0.18-um).

First, we confirmed the fundamental operation of the BCA circuit. We constructed a test circuit that consists of three cells with a periodic boundary condition. As test patterns, we considered sets of initial inputs and desirable outputs such as follows:

$$\frac{U_{j-1}^t U_j^t U_{j+1}^t}{U_j^{t+1}} = \frac{000}{0}, \frac{001}{0}, \frac{010}{0}, \frac{011}{1}, \frac{100}{1}, \frac{101}{1}, \frac{110}{1}, \frac{111}{1}$$

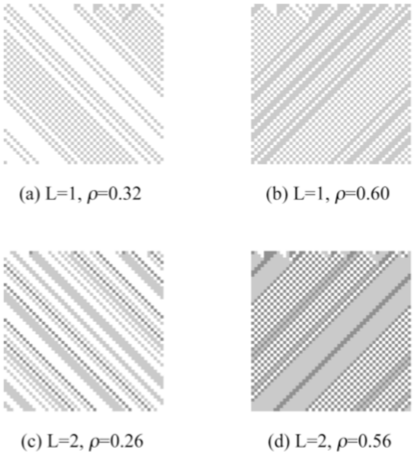

Fig. 6. Time-space diagrams of BCA circuit.

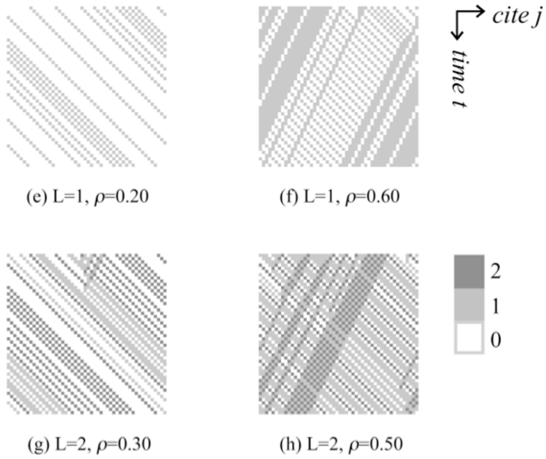

Fig. 7. Time-space diagrams of multi-value SlS CA circuit.

where we assumed L=1. We confirmed other sets of initial inputs and desirable outputs for L=2, 3.

Then, we confirmed the fundamental operation of the multiple-value SlS CA circuit under the same conditions as the above. For instance, we considered a set of initial inputs and desirable outputs as follows:

$$\frac{U_{j-1}^{t-1}U_{j}^{t-1}U_{j+1}^{t-1}U_{j-1}^{t}U_{j}^{t}U_{j+1}^{t}}{U_{j}^{t+1}} = \frac{000000}{0},\ldots,\frac{111111}{1}$$

where we assumed L=1. We confirmed other sets of initial inputs and desirable outputs for L=2, 3.

Figures 6 and 7 show typical time-space diagrams of the BCA circuit and the multiple-value SlS circuit. In both circuits, free flow at low density and congested flow at high density are observed.

V. CONCLUSIONS

We have designed digital VLSI circuits based on the class of the Burgers cellular automata (BCA) for simulating traffic flow: the BCA model and the multiple-value slow-to-start (SlS) CA model [10]. These models are directly related to the Burgers's equation used as macroscopic traffic flow model [14]-[15], and thus these models show macroscopic behaviors such as pattern formation and phase transition observed in real traffic flow [10]. Toward development of efficient traffic simulators, we have designed these models as digital VLSI circuits by assuming a scalable CMOS technology. Through SPICE simulations, we have confirmed the desired operation of these circuits, such as phase transition. The operation speed of the circuits are independent of the number of cells, therefore they are suitable for a large-scale traffic simulation.

In our future work, we are going to implement the BCA model and the multiple-value SlS CA model as quantum LSI systems based on hexagonal BDD approach [17] for a very large-scale simulation of traffic flow.

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