Single-electron circuit as a discrete dynamical system

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Introduction

A distinctive characteristic of a single-electron circuit is that the circuit changes its state discontinuously because of electron tunneling. This enables single-electron circuits to be operated as a discrete dynamical system. In this paper we show an example of such a discrete dynamical system. Our goal is to create novel information-processing devices that use the discrete dynamical behavior of single-electron circuits.

Discrete dynamical system

A discrete dynamical system is a system in which the evolution of the variables is measured in discrete time steps. The behavior of the system is expressed by a difference equation, or a map, that gives the (n+1)th value of variables as a function of the preceding nth value of the variables.

A discrete dynamical system shows complex behavior even if it is a simple one- or two-variable system. An example is the *discrete logistic system* whose behavior is expressed by a map as follows:

$$X_{n+1} = aX_n (1-X_n),$$

shown in Fig. 1(a), where X_{n+1} and X_n are the (n+1)th and nth values of the variable. In this a series of bifurcations occurs when positive parameter a increases; consequently, the system exhibits dynamical behavior from a variety of periodic oscillations to chaos; some examples are shown in Figs. 1(b)-1(d). Other maps similar to the logistic map are illustrated in Fig. 2.

Discrete dynamical behavior in single-electron circuits

The single-electron circuit produces discontinuous change of its state because of electron tunneling, so it can be operated as a discrete dynamical system.

As an example, we consider a single-electron circuit consisting of two coupled single-electron oscillators; Fig. 3(a) shows the configuration of this system. Each oscillator is composed of a tunneling junction C_j and a resistor R_1 or R_2, and it is biased with a positive voltage V_{dd} or a negative one $-V_{ss}$. Coupled by a capacitor C, two oscillators interact with each other and produce the phenomenon of entrainment. The variables of the circuit are node voltages V_1 and V_2. We studied by computer simulation the oscillation and the entrainment in this system. An example of the waveform of node voltage V_2 is shown in Fig. 3(b).

Trajectory and map in the oscillation

The behavior of this single-electron system depends on its circuit parameters. Figures 4(a)-4(c) show the trajectory of the oscillation plotted on a V_1-V_2 plane for three sets of parameters; the trajectory depends on the initial values of V_1 and V_2 but is on an attractor. Figure 4(b) represents the same parameters as those in Fig. 3(b); the flow on the trajectory is 1→2→3→..... →8→1.

Regarding the waveform shown in Fig. 3(b), we define that the discrete-time variable of the system is node voltage V_2 measured just before electron tunneling occurs (or just before a jump of the waveform occurs); the nth value of the variable is denoted by X_n in the figure. By plotting X_{n+1} as a function of X_n, we obtain the map that expresses the behavior of the system. Accordingly, Fig. 5 shows the map for the system that exhibits the behavior shown in Figs. 3(b) and 4(b). We are now developing single-electron systems that exhibit more complex operation and chaotic behavior.

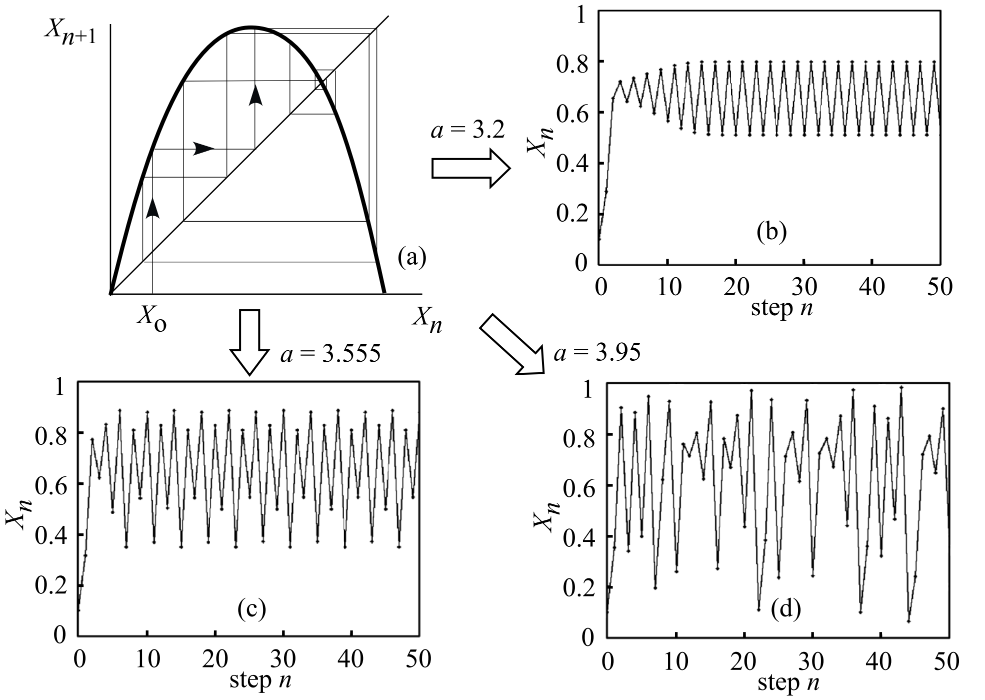

Fig. 1:
Discrete logistic system expressed by $X_{n+1} = aX_n\,(1-X_n)$: (a) map, (b) 2-cycle oscillation, (c) 8-cycle oscillation, and (d) chaotic behavior.

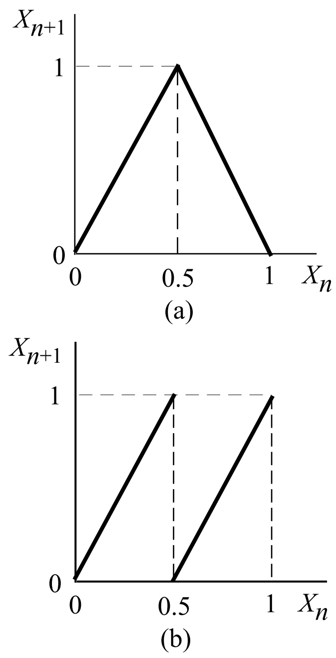

Fig. 2:
Examples of maps: (a) tent map and (b) Bernoulli-shift map.

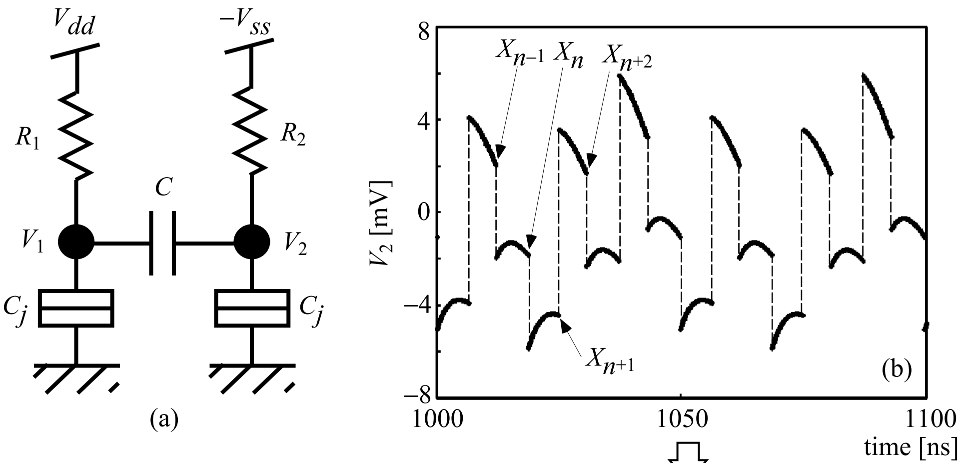

Fig. 3:
Coupled single-electron oscillators: (a) circuit configuration and (b) waveforms of node voltage V_2 for parameters C_j = 10 aF, C = 5 aF, R_1 = 0.4 GΩ, R_2 = 1.2 GΩ, V_{dd} = 12 mV, and $-V_{ss}$ = −12 mV.

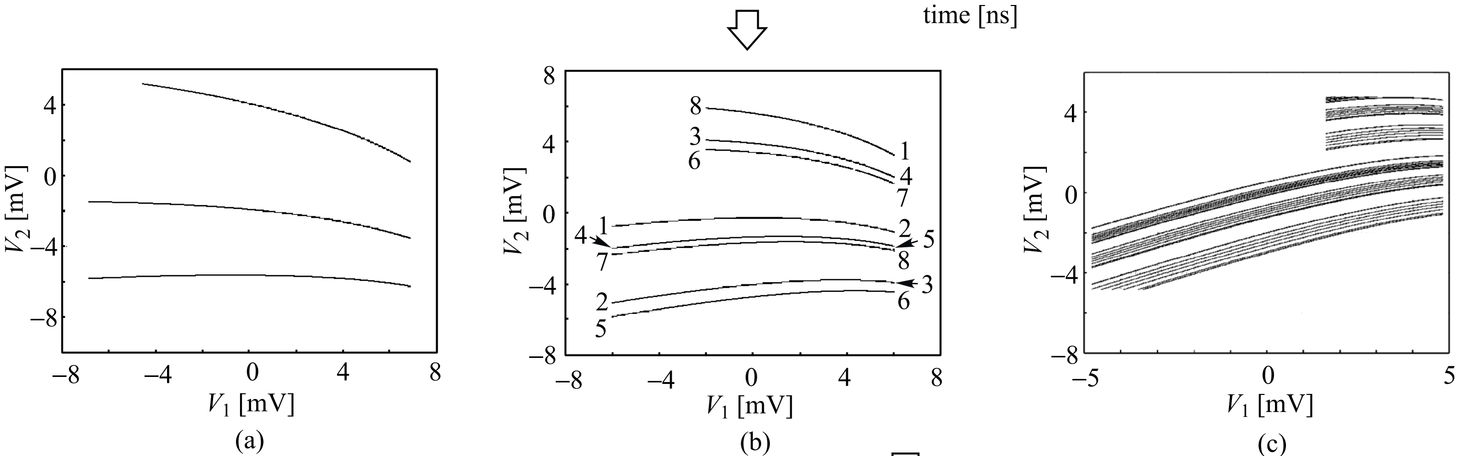

Fig. 4:
Trajectory of the oscillation plotted on a V_1-V_2 plane.
Parameters are the same as given in Fig. 3 (b) except for coupling capacitance C. The value of C is (a) 2 aF, (b) the same as in Fig. 3, and (c) 20 aF.

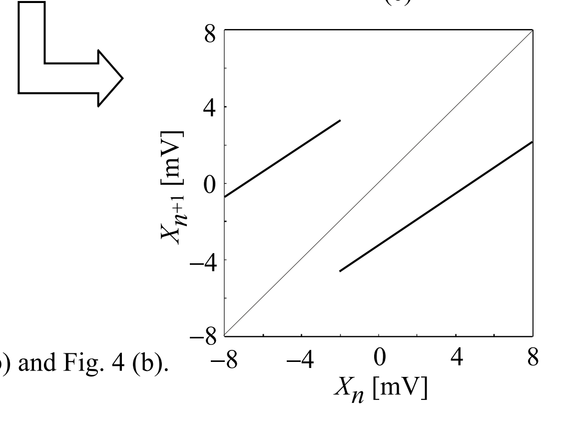

Fig. 5:
The map for the system shown in Fig. 3 (b) and Fig. 4 (b).