

# Neuro-morphic Circuit Architectures Employing Temporal Noises and Device Fluctuations to Improve Signal-to-noise Ratio in a Single-electron Pulse-density Modulator

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We investigated the implications of static noises in a pulse-density modulator based on the Vestibulo-ocular Reflex model. Based on this model, we constructed a simple neuromorphic circuit consisting of an ensemble of single-electron devices and confirmed that static noises (heterogeneity in circuit parameters) and dynamic noises (random noises as a result of spontaneous tunneling events) introduced into the network indeed played an important role in improving the fidelity with which neurons could encode signals whose input frequencies are higher than the intrinsic response frequencies of single neurons. Through Monte-Carlo based computer simulations, we demonstrated that the heterogeneous network could correctly encode signals with input frequencies as high as 1 GHz, twice the range for single (or a network of homogeneous) neurons.

**Keywords:** single-electron devices, neuromorphic LSIs, noise driven LSIs, neural networks.

## 1 INTRODUCTION

Nano-electronic devices are viewed as promising building blocks for the next generation of so-called *Beyond CMOS LSIs*. The *Beyond CMOS devices*

include single-electron devices [1], which operate by regulating the flow of single or a few electrons. Single-electron circuits are thus viewed as potential information processing devices for ultra-low power electronic systems. In addition, because of the high device integration as a result of the minute physical sizes of individual devices, single-electron devices have the potential for applications in parallel-signal processing systems that would require a high density of arrayed devices. In spite of these advantages, single-electron devices suffer from high fabrication mismatches (i.e. variance in individual device parameters), and also have low tolerance to internal and external noises. Therefore to effectively utilize the merits of single-electron devices in creating reliable and efficient electronic systems, there is need to come up with a method to either (i) eradicate these set backs through improved fabrication techniques or compensate for the drawbacks through additional circuitry incorporated into the systems or (ii) effectively utilize these setbacks to create new circuit architectures.

If we look at how neuronal systems function, we find that they have high heterogeneity in intrinsic response properties of individual neurons; they have diverse variances in firing rates, and some of the neurons are even defective. However, in spite of these set backs neurons, as systems, accurately encode signals as they are relayed from sensory organs to the central nervous system, or to other organs. A number of reports suggest that neurons in fact employ heterogeneity to effectively encode signals. Hospedales *et al.* ([3]) demonstrated that neurons in the VOR can encode high frequency signals with a high temporal precision as a result of their heterogeneity.

In this study, toward establishing new circuit architectures for single-electron devices, we investigate the implications of parameter heterogeneity and dynamic noises in reliable transmission of signals in an ensemble of single-electron integrate-and-fire neurons (IFNs). Through Monte-Carlo based computer simulations, we show that heterogeneity in device parameters indeed reduces synchrony among individual neurons, consequently increasing the temporal fidelity with which neurons can encode input signals with frequencies higher than the intrinsic response frequencies of individual neurons.

## **2 PULSE-DENSITY MODULATION IN INTEGRATE-AND-FIRE NEURONS**

An integrate-and-fire neuron (IFN) aggregates inputs from other neurons connected through synapses. The aggregated charge raises the membrane potential until it reaches a threshold, where the neuron fires generating a spike. This spike corresponds to a binary output 1. After the firing event, the membrane potential is reset to a low value, and it increases again as the neuron accepts inputs from neighboring neurons (or input signals) to repeat

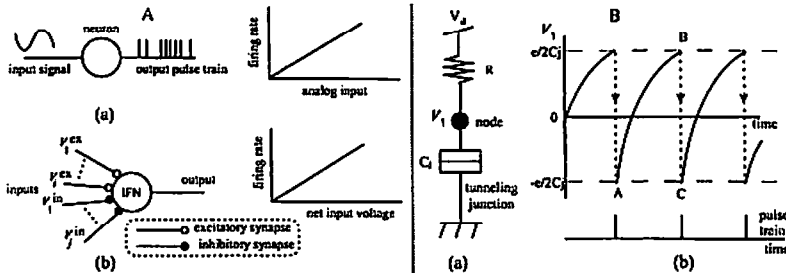


FIGURE 1

A:(a) Pulse density modulation in integrate-and-fire neurons: analog input is converted into a pulse train (b) Fundamental structure and operation of integrate-and-fire neurons (IFNs). The IFN receives input voltages through excitatory and inhibitory synapses, and produces a pulse train whose pulse density (firing rate) is proportional to the net input voltage. B: Single-electron tunneling (SET) oscillator: (a) circuit structure and (b) waveform showing oscillation.

the same cycle; producing a stream of one and zero pulse trains. The spike interval (density of spikes per unit time) is proportional to the analog input voltage i.e. the level of analog input is coded into pulse density. Thus a neuron can be considered as a 1-bit A-D converter operating in the temporal domain. Fig. 1A:(a) shows a schematic representation of analog-to-digital conversion in IFNs. The output pulse density is proportional to the amplitude of the input signal. Fig. 1A:(b) shows the fundamental operation of an IFN. The open circles (o) and shaded circles (●) represent excitatory and inhibitory synapses, respectively. The IFN receives input signals (voltages) through the excitatory synapses (to raise its membrane potential) and inhibitory synapses (which decrease the membrane potential) from adjacent neurons, to produce a spike if the postsynaptic potential ( $\sum V_i^{ex} - \sum V_j^{in}$ ) exceeds the threshold voltage. After the IFN fires, its membrane voltage is reset to a low value, and the integration action resumes.

### 3 SINGLE-ELECTRON OSCILLATOR AS AN INTEGRATE-AND-FIRE NEURON

The operation of an integrate-and-fire neuron (IFN) is modelled with a single-electron oscillator [1] - [2]. A single-electron oscillator (Fig. 1B:(a)) consists of a tunneling junction (capacitance =  $C_j$ ) and a high resistance  $R$  connected in series at the nanodot (●) and biased with a positive or a negative voltage  $V_d$ . It produces self-induced relaxation oscillations if the input voltage is higher than the tunneling threshold ( $V_d > e/(2C_j)$ ) where  $e$  is the elementary charge and  $k_B$  is the Boltzmann constant. The nanodot voltage  $V_1$  increases as the capacitance  $C_j$  is charged through the series resistance (curve AB), until it reaches the tunneling threshold  $e/(2C_j)$ , at which an electron tunnels from the ground to the nanodot across the tunneling junction, resetting the

nanodot voltage to  $-e/(2C_j)$ . This abrupt change in nanodot potential (from B to C) can be referred to as a firing event. The nanodot is recharged to repeat the same cycles. Therefore, a single-electron oscillator could be viewed as an integrate and fire neuron, which aggregates inputs (or inputs from neighboring neurons) producing a pulse when its nanodot voltage reaches the threshold voltage (Fig. 1B:(b)). By feeding a sinusoidal input to a single-electron oscillator, one can adjust the probability of electron tunneling in the circuit: the tunneling rate increases as the input voltage rises above the threshold and gradually decreases to zero as the input approaches and falls below the threshold value. In other words, a single-electron oscillator converts an analog input into digital pulses. A single-electron oscillator can thus be viewed as a pulse-density modulator (PDM), that produces a spike train (or produces zero) if the input signal exceeds (or falls below) the threshold value.

#### 4 MODEL AND CIRCUIT STRUCTURE

The single-electron integrate-and-fire neuron explained in the preceding section is used to study the implications of noises enhancing fidelity of signal transmission in a neuronal single-electron circuit. The circuit is based on a model of the vestibulo-ocular reflex (VOR) proposed by Hospedales *et al.* ([3]). In their work, they reported that noises and heterogeneity in the intrinsic response properties of neurons account for the high-fidelity in VOR functionality.

Fig. 2(a) shows the part of the model, which converts head movements into neural spikes in the VOR, consisting of  $n$  neurons. The structural heterogeneity in the synaptic couplings (membrane time constants) of individual neurons is represented by  $\xi_i$ . We refer to this heterogeneity as static noises. The neurons receive a common analog input and produce spikes whose temporal density

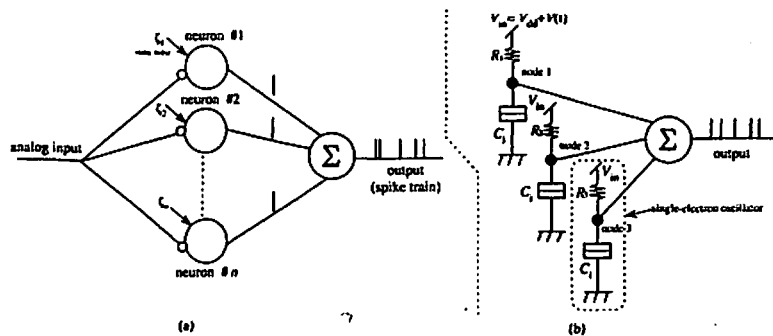


FIGURE 2

(a) Neural network model of signal encoding in the VOR consisting of  $n$  neurons, (b) Implementation with single-electron oscillators.

corresponds to the amplitude of the input signal. The output terminal receives pulses from all the neurons in the network to produce a spike train. The noises introduced into the network lead to random and independent firing events in the neurons, reducing the probability of synchrony in the network. In addition, the variations in parameters increases the randomness with which the network neurons fire, increasing the probability of a ready-to-fire neuron at any given time, which consequently enhances the precision with which the neurons in the network can encode signals with input frequencies higher those of individual neurons.

The network is implemented with single-electron IFNs (oscillators) as shown in Fig. 2(b). The heterogeneity in the model was introduced in the circuit as variations in the series resistance  $R$ . Note that  $R$  is a critical parameter in setting the intrinsic response frequency of each neuron. Therefore, by tuning the values of  $R$ , we could simulate the heterogeneity of membrane time constants of actual neurons.

## 5 SIMULATION RESULTS

In the simulations, all the neurons were connected to an input voltage  $V_{in} = V_{dd} + V(t)$ , where  $V_{dd}$  (bias voltage) was set to 7.8 mV to achieve a monostable operation in the absence of input signals,  $V(t)$  is a pulsed input signal with an amplitude of 0.8 mV. The capacitance of the tunneling junctions  $C_j$  was set to 10 aF. The simulation time was set to 800 ns, while the operation temperature  $T$  was set to 0.5 K.

Fig. 3 shows the transient response of a single neuron. Fig. 3(a) and (c) show the respective input signals with a frequency of 600 MHz and 250 MHz, respectively. Fig. 3(b) shows the neuron response to input "(a)", while "(d)" shows the neuron response to input "(c)". The series resistance was set to  $100M\Omega$ . Fig.3(d) shows successful encoding of the input signal (the neuron fires once for each pulse in the input signal<sup>1</sup>) whose frequency is within the intrinsic firing rate of a single neuron. In Fig. 3(b), the neuron could only encode some of the input pulses, leading to a lower firing rate as compared to the input rate. In other words, the neuron in (b) could only transmit some of the input pulses toward the output. This degrades the fidelity of signal transmission along the neural network.

Fig. 4 shows the response of a single neuron over a wide range of input frequencies. The horizontal axis shows the input frequency, while the vertical axis shows the average firing rate of the neuron. The neuron response

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<sup>1</sup> Tunneling (firing) in single-electron devices involves a probabilistic time lag or waiting time between when the node voltage exceeds the threshold voltage and when an electron can actually tunnel from the ground to the node, releasing a spike toward the output terminal. Due to the effect of the time lag, a neuron might fail to fire even after achieving the tunneling conditions as seen in Fig. 3(d). As a result, the average firing rate would be somewhat lower than the input pulse rate

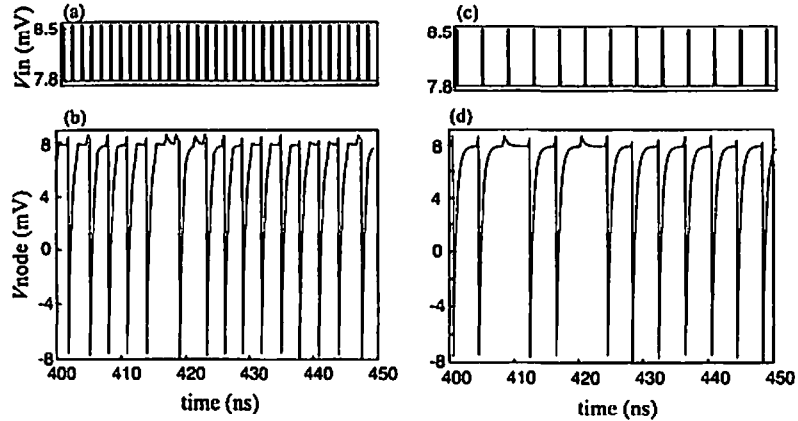


FIGURE 3

Transient response of a single neuron. (a) and (c) show input signals with input frequencies of 600 MHz and 250 MHz, respectively. (b) and (d) show the output characteristics of neurons fed with input signals of 600 MHz and 250 MHz, respectively.

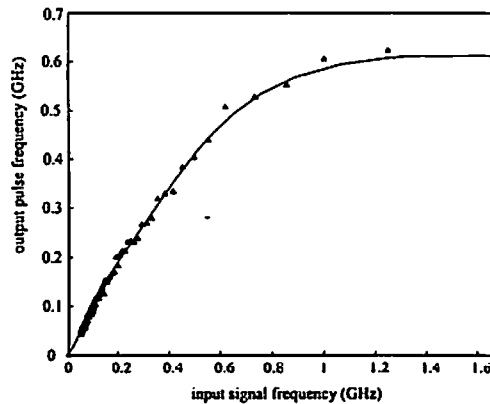


FIGURE 4

Output firing rate of a single neuron plotted against the input pulse frequency. Firing rate saturates for input frequencies beyond 600 MHz.

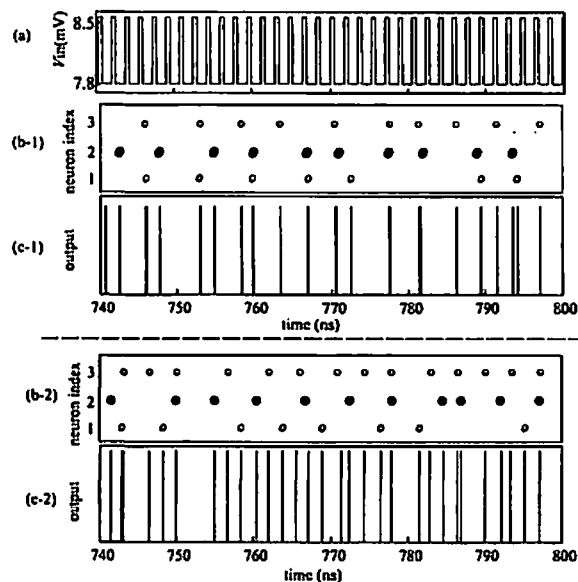
was linear for input signals with a frequency of upto 500 MHz. Beyond this range, the output was highly distorted. This shows that a single neuron can successfully encode (respond to) signals with a maximum input frequency of 500 MHz.

The response of a population of neurons to various input frequencies was investigated with two sets of neuron ensembles: homogeneous and heterogeneous networks. In the homogeneous ensemble, the series resistances  $R_1$ ,  $R_2$ ,

and  $R_3$  were set to the same value, whereas in the second set, heterogeneity (static noises) was introduced by varying the values of series resistances in the three neurons. The results are shown in Figs. 5 and Fig. 6.

Fig. 5(a) shows the input signal with a frequency of 600 MHz. Figs. 5(b-1) and (c-1) show the response of the homogeneous network, where the series resistances  $R_1$ ,  $R_2$  and  $R_3$  were set to  $100\text{ M}\Omega$ . Fig. (b-1) shows the firing events of individual neurons in the network. Fig. (c-1) shows the summed spike output (spike train) at the output terminal. We could confirm that the neurons in the homogeneous network tend to synchronize, emitting pulses at almost the same timing.

Figs. 5 (b-2) and (c-2) show the response of neurons in the heterogeneous network, where the series resistances were set to  $110\text{ M}\Omega$  for neuron 1,  $100\text{ M}\Omega$  for neuron 2 and  $90\text{ M}\Omega$  for neuron 3. The firing events in the heterogeneous network are more or less random as shown in Fig. 5(b-2). The probability of having a neuron with a potential near the threshold value, at any given moment, is higher than in the case of a homogeneous network. Thus the network can respond to any incoming pulses at a higher probability. This results in an improved encoding of the input as illustrated by the spike train shown in Fig. 5(c-2). In other words, since the neurons fired irregularly, they



**FIGURE 5**

Transient responses of both homogeneous and heterogeneous networks. (a) shows the input signal. (b-1) shows the firing events of each neuron, while (c-1) shows the summed pulse output for the three neurons in the homogeneous network. (b-2) shows the firing events, and (c-2) shows the summed pulse output of the heterogeneous network.

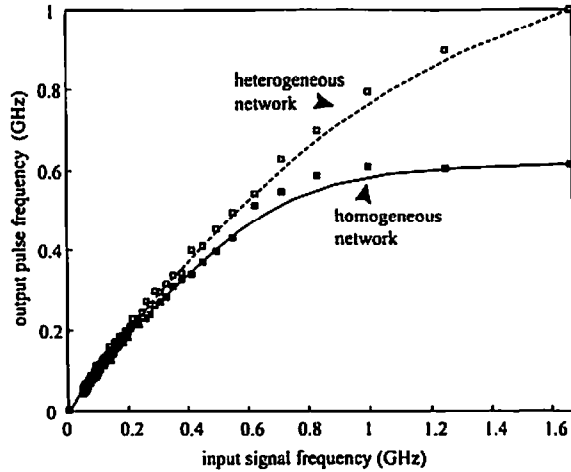


FIGURE 6

Output firing rate of an ensemble of neurons plotted against the input pulse frequency. The homogeneous network can encode signals of upto 500 MHz, as compared to operation range of 1 GHz for the heterogeneous network.

could transmit the input pulses with a higher temporal precision as opposed to the homogeneous network. This is elaborated in more detail in Fig. 6, where the transmission of signal over a wide range of frequencies is demonstrated. The horizontal axis represents the frequency of input signals, while the vertical axis shows the average firing rate (output frequency) for both neuron sets. In the case of the homogenous network, since the neurons tend to synchronize with time, their encoding frequency is the same as that of individual neurons. Contrary, neurons in the heterogeneous network could correctly encode signals with input frequencies upto 1 GHz, twice that of the homogeneous network. This demonstrates that heterogeneity in the circuit parameters (presence of static noises) plays an important role in improving the fidelity with which neurons can encode signals with input frequencies far beyond the encoding capacity of individual neurons.

## 6 EFFECT OF DYNAMIC NOISES

Hospedales *et al.* ([3]) investigated the importance of random noises in improving the fidelity of signal transmission in the VOR. They concluded that besides neuronal heterogeneity, externally induced noises also play an important role in improving the network performance. These external noises could be as a result of spontaneous increases or decreases of membrane potential due to firing events in other neurons in the network. These changes are random and are often referred to as dynamic noises. In our circuit, we studied the



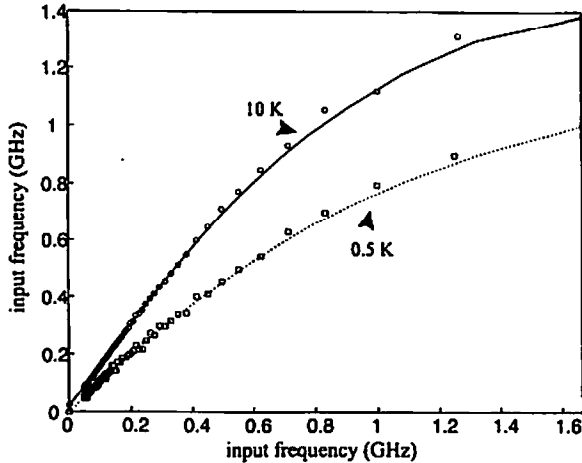


FIGURE 7

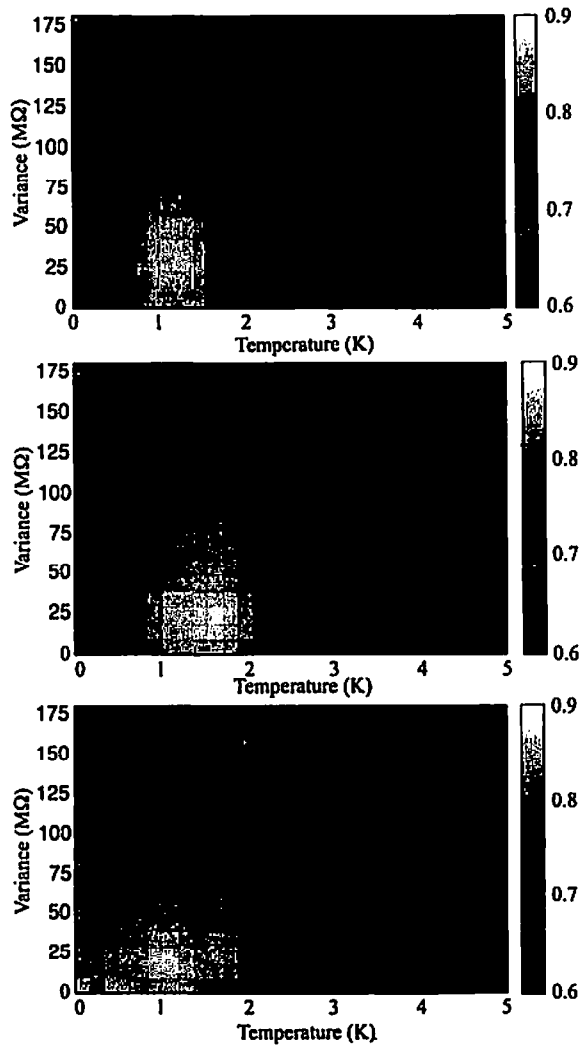
Output firing rate of an ensemble of neurons plotted against the input pulse frequency for temperature  $T = 0.5$  and  $10$  K. Dynamic noises as a result of increased temperature compensate for the roll-off at higher temperatures.

effect of dynamic noises by considering thermally induced tunneling events in the network. Fig. 7 shows the response characteristics of a network simulated at  $0.5$  K, and  $10$  K. As the temperature increases, thermally induced tunneling events in single-electron neurons increase, resulting in an increase in the average firing rate in the network. This is illustrated by the increased firing rate at a temperature of  $10$  K. Although this work suggests that dynamic noises don't play a critical role in increasing the maximum response frequency of the network, they however, increase the fidelity with which the network can sample input signals within the maximum input signal frequency range determined by heterogeneity in the network elements. This is evident at higher input frequencies, where the ratio of the output pulse rate to the input pulse rate starts to roll-off rapidly. The roll off is compensated for by the dynamic noises, which reduces the effect of waiting time in electron tunneling.

## 7 EFFECT OF DYNAMIC AND STATIC NOISES

To study the effect of both noises in the transmission fidelity of a heterogeneous network, we calculated the correlation between the input and the output signals in a network of  $100$  noisy neurons. The neurons were fed with a bias voltage of  $8$  mV, a sinusoidal input signal with a frequency ( $f$ ) of  $500$ ,  $400$  and  $200$  MHz and peak-to-peak amplitude of  $1$  mV above the bias voltage. The simulation results are shown in Fig. 8 where  $f = 500$  MHz for (a),  $f = 400$  MHz for (b), and  $f = 200$  MHz for (c). The horizontal axis shows the

operation temperature ( $T$ ), while the vertical axis shows the variance ( $\sigma$ ) of the series resistances with a mean value of 100 MHz. The scale of the color grading is shown on the right, with the light shading representing a correlation value of 0.9, and the dark shading representing a correlation of 0.6. From the results we observe that the network could produce the maximum correlation value ( $C_{\max}$ ) of 0.87 at a variance of 30 MHz and a temperature of



**FIGURE 6**

Effect of static (variance) and dynamic noises (temperature) to the correlation values in an ensemble of neurons fed with various input signal frequencies: 500 MHz for (a), 400 MHz for (b) and 200 MHz for (c).

1.25 K, for the given set of circuit parameters and input signal frequency of 500 MHz,  $C_{\max}$  of 0.89 at a variance of 25 MHz and a temperature of 1.75 K at an input frequency of 400 MHz, and  $C_{\max}$  of 0.88 at a variance of 20 MHz and a temperature of 1.25 K at an input frequency of 200 MHz. This confirms that the effect of static noises is more dominant in enhancing the fidelity of transmission of high-frequency input signals.

## 8 CONCLUSION

In this study, we proposed and investigated the implication of heterogeneity in transmission of high frequency signals in a neural network. Through Monte-Carlo based computer simulations, we confirmed that heterogeneity in device parameters indeed improved the temporal precision with which the network could transmit signals with high input frequencies within the network. A heterogeneous network could correctly encode signals of up to 1 GHz, as compared to 500 MHz in single neurons (or a network of homogeneous neurons). Another important factor to consider in improving the fidelity of this circuit would be the effect of external and internal (dynamic) noises. In single-electronic devices, such noises include thermally induced random firing events or the effect of environmental noises. As we have shown, as the temperature increases, the dynamic noises also increase, compensating for the roll-off in response of the network, especially at high frequencies. Although a comprehensive investigation on the implications of dynamic noises to signal transmission is required, the preliminary results presented in this paper show that in addition to heterogeneity in neuron properties, externally introduced noises could assist in further improving the fidelity of signal encoding in single-electron circuits. We should however, note that at higher temperatures, beyond the results presented here, random tunneling as a result of dynamic noises would increase rapidly leading to degradation of signal transmission. Therefore, the value of dynamic noises to be introduced to the network to achieve the best performance needs to be optimized.

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