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Excitable Reaction-Diffusion Media with Memristors

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Abstract

We numerically investigated the dynamics of new reaction-diffusion-type excitable media where the diffusion coefficient is represented by *memristive* dynamics. The medium consists of an array of excitable Oregonators and each Oregonator is locally coupled with each other via *memristors*, which were claimed to be the fourth circuit element exhibiting a relationship between flux ϕ and charge q. Through extensive numerical simulations, we found that i) memristor conductances were modulated by excitable waves passing on the memristor depending on the memristor's polarity, ii) the velocity of the excitable wave propagation is thus modulated by the change in the memristor conductance, and the degree of modulation is inversely proportional to the time constant of the memristor model, and iii) nonuniform spatial patterns are generated under the periodic boundary condition (ring structure of excitable media).

1. Introduction

Semiconductor reaction-diffusion (RD) computing LSIs implementing RD dynamics have been proposed in the literature [1]. These LSIs were mostly designed by digital, analog, or mixed-signal complementary-metal-oxide-semiconductor (CMOS) circuits of cellular neural networks (CNNs) or cellular automata (CAs). Electrical cell circuits were designed to implement several CA and CNN models of RD systems [2, 3, 4, 5, 6], as well as fundamental reaction-diffusion equations [7, 8, 9, 10]. Each cell is arranged on a two-dimensional (2-D) square or hexagonal grid and is connected with adjacent cells through coupling devices that transmit cell's state to its neighboring cells, as in conventional CAs. For instance, an analog-digital hybrid RD chip [3] was designed for emulating a conventional CA model for Belousov-Zhabotinsky (BZ) reactions [11]. A full-digital RD processor [4] was also designed on the basis of a multiple-valued CA model, called *excitable lattices* [12]. An analog cell circuit was also designed to be equivalent to spatial-discrete Turing RD systems [8]. A full-analog RD chip that emulates BZ reactions has also been designed and fabricated [7].

Blueprints of non-CMOS RD chips have been designed; *i.e.*, a single-electron RD device [13]. The authors previously proposed a RD device based on minority-carrier transport in semiconductor devices [14]. The point of our idea was to simulate chemical diffusion with minority-carrier diffusion in semiconductors and autocatalystic chemical reactions with carrier multiplication in *p-n-p-n* negative resistance diodes. Using CMOS and non-CMOS RD circuits enables us to simulate a variety of autocatalytic reactions and open up a variety of application fields for RD devices.

In this paper, we try to reclaim a new field of semiconductor RD LSIs, and propose a new RD-based excitable medium, keeping in mind the hardware implementation. Recently, so-called "memristors", originally introduced by Leon Chua in 1971 [15] and claimed to be the fourth circuit element exhibiting a relationship between flux ϕ and charge q, have been re-spotlighted since Strukov *et al.* presented equivalent physical examples [16]. Although the presented device was a bipolar resistive RAM that did not "directly" exhibit a relationship between ϕ and q, the device behaved as a non-volatile resistor whose resistance was continuously controlled by the amount of the charge flow (current). Here a question arises: What happens if one replaces resistors for diffusion in analog RD LSI with memristors? This is the primary purpose of the investigation in this work. In the following sections, we introduce an excitable RD model with memristors, and show the spatiotemporal behaviors of 1-D RD models through extensive numerical simulations.

2. The Model

A general model of memristors is explained in terms of *memristance* $M(q)$ [15]; however, we here use a comprehensive model represented by

$$i = g(w)v, \quad \frac{dw}{dt} = i \tag{1}$$

where v represents the voltage across the memristor, i the current of the memristor, w the nominal internal state of the

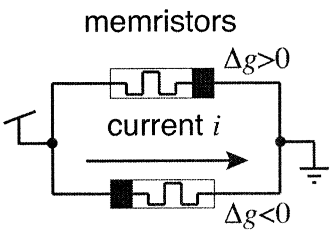


Figure 1: Memristor symbols and polarity definition

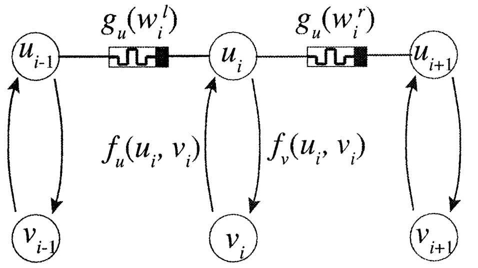


Figure 2: Electrical representation of RD system whose diffusive resistors are replaced with memristors

memristor and corresponds to the charge flow of the memristor, and $g(w)$ the monotonically increasing function with increasing w [16]. This model implies that positive (or negative) i (current flow) increases (or decreases) w, which results in an increase (or decrease) in the memristor conductance $g(w)$. Figure 1 illustrates these aspects of memristors, where Δg corresponds to dw/dt and hence $dg(w)/dt$. In the following, we integrate these dynamics into a general RD model.

A 1-D reaction-diffusion system is described by

$$\frac{\partial u(x)}{\partial t} = g_u \nabla^2 u(x) + f_u[u(x), v(x)] \quad (2)$$

$$\frac{\partial v(x)}{\partial t} = g_v \nabla^2 v(x) + f_v[u(x), v(x)] \quad (3)$$

where $u(x)$ and $v(x)$ represent the concentration at spatial position x, $g_{u,v}$ the diffusion coefficients, and $f_{u,v}(\cdot)$ the reaction model. We here employ Oregonators [17] for the reaction model; *i.e.*,

$$\begin{aligned} f_u[u(x), v(x)] &= u(x)[1 - u(x)] - av(x)\frac{u(x) - b}{b + u(x)} \\ f_v[u(x), v(x)] &= u(x) - v(x) \end{aligned}$$

and consider the excitable properties ($g_v = 0$ only). Although g_u is constant in general RD models, we are interested in a system where g_u is locally modified by the potential gradient of $u(x)$.

When $u(x)$ and $v(x)$ are represented by voltages on RD hardware, the gradient (diffusion terms in the RD model) is represented by linear resistors [7]. For example, if one discretizes Eq. (2) spatially as

$$\frac{du_i}{dt} = \frac{g_u(u_{i-1} - u_i) + g_u(u_{i+1} - u_i)}{\Delta x^2} + f_u(\cdot)$$

where i is the spatial index and Δx the discrete step in space, the terms $g_u(u_{i-1} - u_i)$ and $g_u(u_{i+1} - u_i)$ represent currents flowing into the i-th node from the $(i-1)$-th and $(i+1)$-th nodes via two resistors whose conductances are represented by g_us divided by Δx^2. Here we introduce the memristor model described by Eq. (1); *i.e.*, the resistors are replaced with memristors. The resulting point dynamics are

$$\begin{aligned} \frac{du_i}{dt} &= \frac{g_u(w_i^l)(u_{i-1} - u_i) + g_u(w_i^r)(u_{i+1} - u_i)}{\Delta x^2} + f_u(\cdot) \\ \frac{dv_i}{dt} &= f_v(\cdot) \end{aligned}$$

where $g_u(\cdot)$ represents the monotonically increasing function defined by

$$g_u(w_i^{l,r}) = g_{\min} + (g_{\max} - g_{\min}) \cdot \frac{1}{1 + e^{-\beta w_i^{l,r}}}$$

where β represents the gain, $g_{\min,\,\max}$ the minimum and maximum coupling strengths, and $w_i^{l,r}$ the variables for determining the coupling strength of the i-th Oregonator (l: leftward, r: rightward). Finally, we introduce the following memristive dynamics for $w_i^{l,r}$:

$$\tau \frac{dw_i^{l,r}}{dt} = g_u(w_i^{l,r}) \cdot (u_{i-1,i+1} - u_i)$$

where the right side represents the current of the memristors in Eq. (1). The model above corresponds to an electrical RD system consisting of Oregonators whose diffusive resistors are replaced with memristors (Figure 2).

3. Simulation Results

In the following simulations, we use the following parameters for memristors: $\beta = 1$, $g_{\min} = 10^{-4}$, and $g_{\max} = 10^{-1}$.

First, we simulated the basic model shown in Figure 3(a). One side of the boundary was stimulated by a periodic pulse sequence, and we measured the conductance of the memristor. The initial conductance of the memristor was set at zero. Figure 3(c) shows the simulated results. The conductance was increased greatly during the onset of the input pulse, which resulted in a small increase in the conductance. We roughly estimated Δg per single pulse (0.17 mS / pulse). Figure 3(b) shows the opposite simulation setup. In this simulation, the polarity of the memristor was inverted, so one can expect the conductance to be decreased by the input pulses. The initial conductance was chosen so that stimulations to u_0 can cause chain excitation on u_1 via the memristor. Figure 3(d)

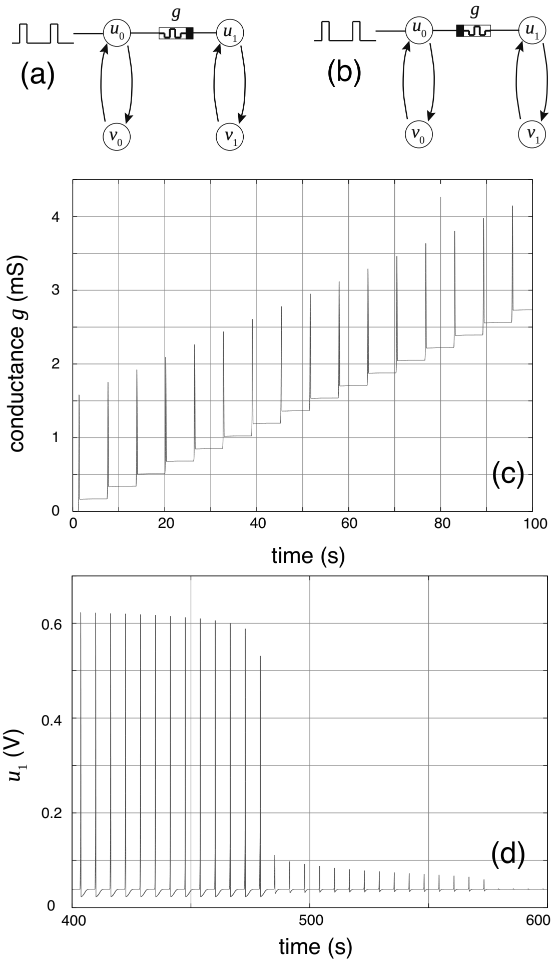
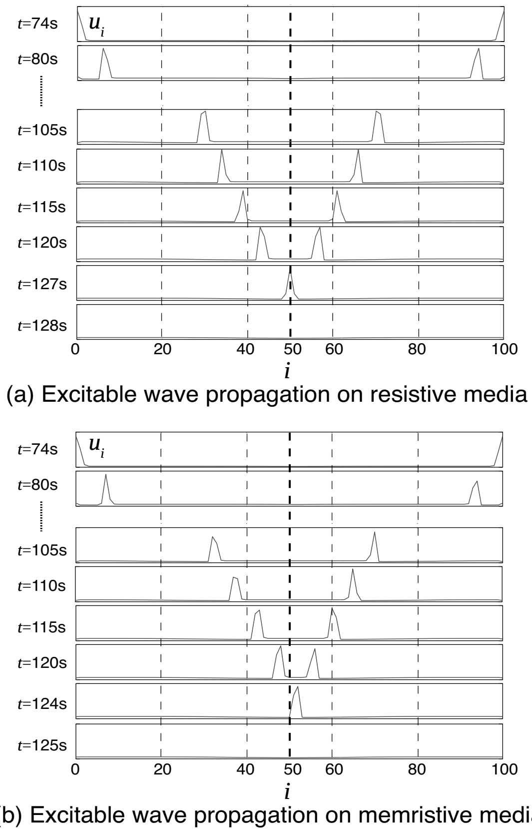


Figure 3: Simulation results for two Oregonators with memristors

shows the temporal responses of u_1. The stimulus was initially applied (u_1 was excited), but was terminated because of the decrease in the conductance.

Figure 4(a) shows simulation results of a 1-D medium with 100 Oregonators without memristive effects. Excitable wave propagation on the medium is apparent. Both boundaries were simultaneously stimulated, and the waves collided at the center position (then they disappeared). When we introduced memristive effects here, since coupling strength $g_u(w_i^{l,r})$ is modified by the direction of wave propagation, the results were different from those in Figure 4(a). Figure 4(b) shows simulation results of a 1-D medium consisting of 100 Oregonators with memristive effects, where the velocity of each excitable wave was different depending on the direction of wave propagation, which resulted in wave collision at a non-center position. Excitable waves moving rightward (in the figure) increased $w_i^{l,r}$ of memristors under the wave, whereas leftward waves decreased $w_i^{l,r}$ under the wave, as a result of



(a) Excitable wave propagation on resistive media

(b) Excitable wave propagation on memristive media

Figure 4: Simulation results for 100 Oregonators consisting of normal resistors (a) and memristors (b)

the polarity of memristors shown in Figure 1.

Finally, we prepared a 1-D medium with 100 Oregonators as well, and assumed a cyclic boundary condition. After stimulating one node, an excitable wave propagates on the medium in a cyclic-looping manner. The initial stimulation was aware of the unidirectional wave propagation by controlling the refractory states of the Oregonators. Figure 5 shows time courses of all the nodes (u_i) where the magnitudes are represented by gray-scale tones. With time, spatial (nonuniform) patterns developed. Surprisingly, the developed pattern was periodic, like Turing patterns, and it reached equilibrium at around 2×10^4 iterations.

4. Summary

We proposed a new reaction-diffusion-based excitable medium that employed *memristors* to represent the diffusion coupling. Through numerical simulations, we found that the medium could develop spatial patterns, and the equilibrium pattern was periodic. We will further investigate the spatiotemporal properties of i) a model having random replacement (polarity) of memristors and ii) a 2-D version of the model.

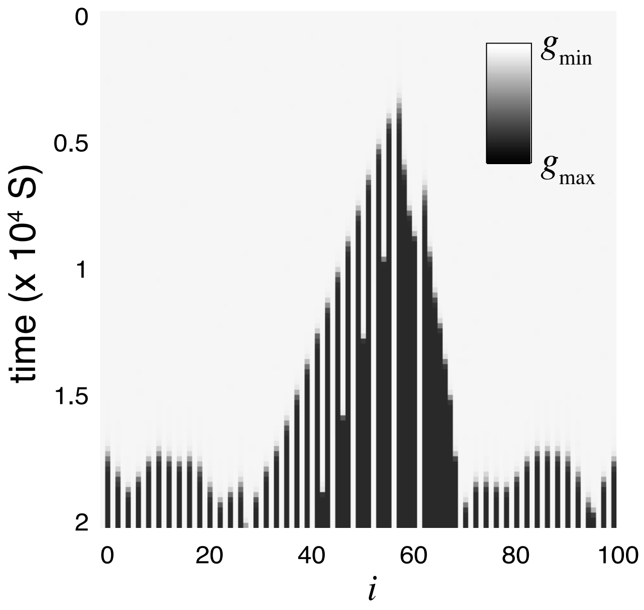


Figure 5: Spatial pattern formation on 1-D excitable media with memristors under cyclic boundary condition

Acknowledgments

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