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Analog computation using quantum-flux parametron devices

Takatoshi Inokuchi, Takashi Yamada, Tetsuya Asai, Yoshihito Amemiya *

Department of Electrical Engineering, Hokkaido University, Kita 13, Nishi 8, Sapporo 060-8628, Japan

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Abstract

Analog computation is a processing method that solves a given problem by utilizing an analogy of a physical system to the problem. An idea is presented here for relating the behavior of quantum-flux parametron circuits to analog computation. As an example, a method is proposed for solving a combinatorial optimization problem, the max-cut problem, by utilizing the properties of quantum-flux parametron circuits. In problem solving, a parametron circuit is constructed whose free energy is related to the objective function of a given problem and then is made to settle down to its minimum energy state. The solution to the problem can be obtained by checking the final state that the circuit reaches. The effectiveness of this method was confirmed by computer simulation for sample problem instances.

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1. Introduction

One promising research area in single-flux-quantum electronics is the development of novel computation devices based on non-Boolean logic architectures. We here propose one such device, an analog computation device using single-flux parametron circuits.

Analog computation [1,2] is a processing method that solves a mathematical problem by applying an analogy of a physical system to the problem. To solve the problem with this method, one prepares an appropriate physical system and represents each problem variable by a physical quantity in the system. If the mathematical relations between the physical quantities are analogous to those of the problem, then one can find the solution to the problem by observing the behavior of the system and measuring the corresponding physical quantities. In this computation, a given problem is mapped onto the physical system and is solved through concurrent or parallel operation of all the elements in the system. Through this parallelism, analog computation can provide the possibility of solving complex problems in a short time regardless of the size of the problem.

Proposed here is an analog computation device that solves combinatorial-optimization problems by utilizing the energy-minimizing property of quantum-flux parametron circuits. By constructing a parametron circuit whose free energy is related to the objective function of a given combinatorial

* Corresponding author. Tel.: +81-11-706-6080; fax: +81-11-706-7890.
E-mail address: amemiya@sapiens-ei.eng.hokudai.ac.jp (Y. Amemiya).

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problem, we can solve the problem simply by observing to what state the circuit will settle down. The concept of this device is explained in the following sections with an example of a typical combinatorial-optimization problem, the max-cut problem.

2. Max-cut problem

The max-cut problem is stated as follows: Given a graph $G = (V, E)$ with positive weights on the edges, find a partition of the vertices $V = \{1, 2, \ldots, n\}$ into disjoint sets V_0 and V_1 such that the sum of the weights of the edges that have one endpoint in V_0 and one endpoint in V_1 is maximal. As an instance, a weighted graph and its maximal cut are shown in Fig. 1.

To formulate the objective function for the max-cut problem, we here define a number of variables. Let d_{ij} be the weight associated with the edge $\{i, j\}$ (by definition, $d_{ij} = d_{ji}$) and let x_i be a $1/-1$ variable defined as

$$\begin{aligned} x_i &= 1 \quad (\text{if } i \in V_1) \\ x_i &= -1 \quad (\text{if } i \in V_0) \end{aligned} \tag{1}$$

then the max-cut problem can be formulated as

$$\text{maximize} \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{d_{ij}}{4}(x_i - x_j)^2, \tag{2}$$

which can be rewritten as

$$\text{minimize} \; -\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} x_i x_j. \tag{3}$$

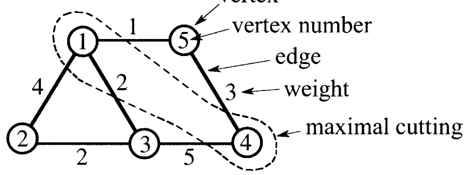

Fig. 1. An instance of a weighted graph. The maximal cut is given by two vertex sets {1, 4} and {2, 3, 5}.

3. Quantum-flux parametron

The quantum-flux parametron [3] is a switching device (Fig. 2) consisting of two Josephson junctions J_1 and J_2, load inductor L_d, and two excitation transformers consisting of primary inductors L_p and secondary inductors L_s. It has two superconducting loops, i.e., left loop J_1–L_s–L_d–J_1 and right loop J_2–L_s–L_d–J_2. To operate the parametron, we send the excitation current I_{ex} into the primary inductors, thereby applying magnetic flux to each loop. For small values of this excitation flux, the parametron is in a monostable state and almost no flux threads the loops. If the excitation flux increases to exceed a critical value (about a quarter of the fluxoid quantum Φ_0, the parametron turns into a bistable state and a net flux begins to thread either loop. The threading net flux increases with increase in the excitation flux and reaches nearly Φ_0 when the excitation flux increases to $\Phi_0/2$. We here define the parametron is in state “1” if the net flux is threading the left loop and is in state “−1” if the net flux is threading the right loop. The state can be identified by observing the direction of the current flowing in the load inductor L_d, the current flows downward to GND in state “1” and upward from GND in state “−1”. Which of the states the parametron takes depends on the polarity of a seed flux applied to the load inductor from the outside of the parametron.

4. Analogous circuit for the max-cut problem

Using quantum-flux parametrons, we can construct an analogous circuit whose free energy is related to the objective function of the max-cut

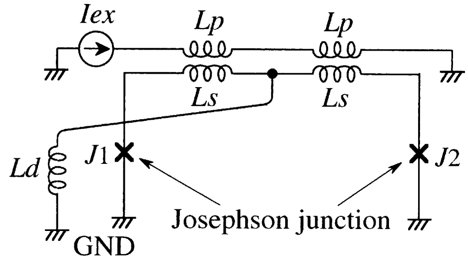

Fig. 2. Quantum-flux parametron.

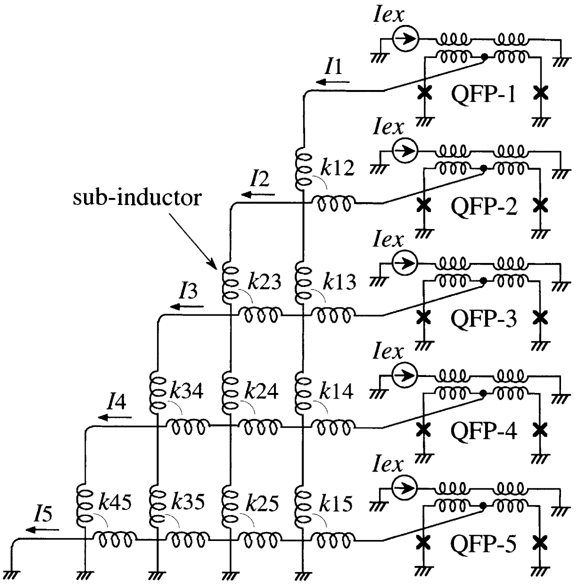

Fig. 3. Quantum-flux parametron circuit for solving the max-cut problem. The circuit for a five-vertex graph is shown.

problem. Taking a problem graph having five vertices as an example, the configuration of the analogous circuit is shown in Fig. 3. We prepare five parametrons (QFP-1 through QFP-5) and represent vertex i of the problem graph by ith parametron. To represent edge $\{i,j\}$ with weight d_{ij}, we magnetically couple ith parametron with jth parametron $(i,j = 1,2,3,4,5)$; i.e., we divide the load inductor of each parametron into four identical inductors as shown in Fig. 3 and couple magnetically a divided load inductor of ith parametron with that of jth parametron by mutual induction coefficient k_{ij}. The value of k_{ij} $(|k_{ij}| < 1)$ is set at a negative value proportional to $-d_{ij}$. In this way, we can construct the analogous circuit for any problem graph given.

To solve the problem, we turn on excitation current I_{ex} for the parametrons and increase it continuously. When the excitation current, therefore the excitation flux, exceeds a critical value, the state of all the parametrons becomes bistable; consequently, some parametrons settle down in state "1" and the others settle down in state "-1". According to their $1/-1$ states, we partition the vertices of the problem graph into two sets, i.e., the vertices corresponding to "1"-state parametrons and the vertices corresponding to "-1"-state parametrons. The maximal cut for the problem graph is given by these two sets of vertices.

This solution is based on the energy-minimizing behavior of the parametron circuit. The free energy of the circuit is given as follows. We here consider a circuit consisting of n parametrons and define variables x_i $(i = 1,2,\ldots,n)$ by

$$x_i = 2(\Phi_i - \Phi_{ex})/\Phi_0, \tag{4}$$

where Φ_{ex} is the excitation flux induced by the excitation current, and Φ_i is the net flux threading the left loop of ith parametron. Variable x_i represents the state of ith parametron; it is 1 or -1 for $\Phi_{ex} = \Phi_0/2$ because, as mentioned in the previous section, the net flux in the loop is nearly Φ_0 or zero for $\Phi_{ex} = \Phi_0/2$. Using variables x_i, the free energy F of the circuit is given by

$$F = \left[\frac{\Phi_0^2}{2(n-1)L_d}\right]\left[\left\{\frac{n-1}{4}\right\}\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}x_i x_j + \left\{\frac{2(n-1)L_d I_0}{\Phi_0}\right\}\left\{\frac{1}{\pi}\cos\left(\frac{2\pi\Phi_{ex}}{\Phi_0}\right) - \frac{2\Phi_{ex}}{\Phi_0}\sin\left(\frac{2\pi\Phi_{ex}}{\Phi_0}\right)\right\} \times \sum_{i=1}^{n}\{1-\cos(\pi x_i)\}\right], \tag{5}$$

where L_d is the total load inductance of each parametron, I_0 is the maximum supercurrent of the Josephson junctions, and k_{ij} is the mutual induction coefficient between the sub-inductors of ith and jth parametrons. It was assumed here that $\Phi_0 \gg 2\pi L_s I_0$ and $|k_{ij}| \ll n$. Note that the second term in Eq. (5) is identical to the objective function (Eq. (3)) of the max-cut problem.

To operate the circuit successfully, L_d and I_0 have to be set at values such that $4\pi L_d I_0 > \Phi_0$. Under this condition, we turn the excitation current on and increase it, thereby increasing excitation flux Φ_{ex} from 0 to $\Phi_0/2$. In zero excitation $(\Phi_{ex} = 0)$, the free energy is minimum at $x_i = 0$ $(i = 1,2,\ldots,n)$ because the third term is dominant; consequently, the circuit takes this zero state. If the excitation current is started and Φ_{ex} is thereby increased to $\Phi_0/2$, the third term is dominant again and therefore the free energy becomes minimum at $x_i = 1$ or -1 $(i = 1, 2, \ldots, n)$; there are 2^n possible minimum states (2^n combinations of x_i) and the circuit changes from the initial zero state to one of these 2^n states. To which of the states the circuit changes is determined during the increase in Φ_{ex}. When Φ_{ex} increases to a critical value (about $\Phi_0/4$), the first and the third terms in Eq. (5) offset each other at around $x_i = 0$. In this condition, the second term becomes dominant and, consequently, the circuit begins to change its state so that the second term, identical to the objective function of the problem, will become minimum.

5. Problem-solving operation and discussion

For various instances of the max-cut problem, we confirmed the problem-solving operation of the analogous circuit by computer simulation. Illustrated here is a result for the problem instance given in Fig. 1. The analogous circuit used has the same configuration as shown in Fig. 3. The circuit parameters were: $n = 5$, $L_d = 24$ pH, $L_s = 2$ pH, and $I_0 = 0.05$ mA. The mutual inductance between the primary inductor and the secondary inductor in the excitation tansformer was assumed to be 1.6 pH. For every Josephson junction, parallel resistance of 5 Ω and parallel capacitance of 0.1 pF were assumed. The mutual induction coefficient k_{ij} between each two divided load inductors was set at $-0.1 \times d_{ij}$, where d_{ij} is the weight of the edge given in Fig. 1.

The result is shown in Fig. 4. We retrieved the state of parametrons by observing the direction of the current (I_1 through I_5) in the load inductors. In this example, the excitation current was gradually increased from 0 to 0.45 mA with a time constant of 20 ns. When the excitation current increased to exceed the critical value, all the parametrons turned into a bistable state as shown in the figure. Their final states represent the solution to the problem; i.e., the maximal cut for the problem graph is given by two vertex sets {1, 4} and {2, 3, 5}. We repeated this solving procedure many times and confirmed that every trial resulted in the correct solution. Thus we can find the solution to the max-cut problem by using the quantum-flux parametron circuit. This solving method can be used for solving other combinatorial problems because many combinatorial problems can be reduced to the max-cut problem.

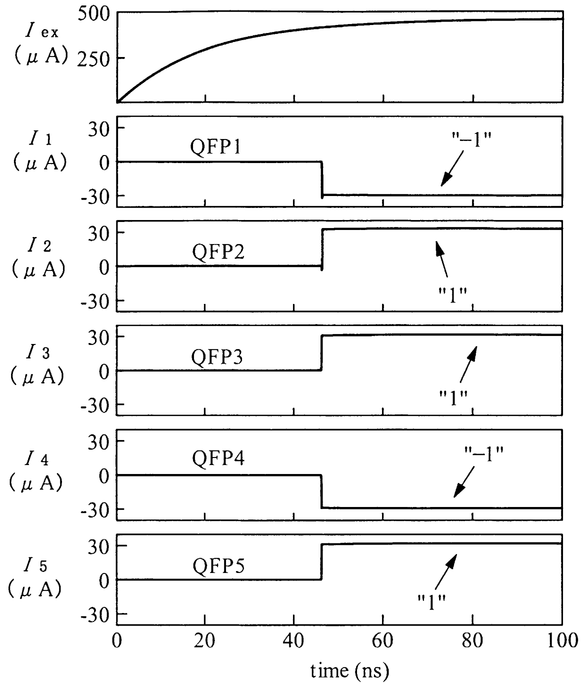

Fig. 4. Problem-solving operation of the quantum-flux parametron circuit (simulation). I_{ex}: excitation current, I_1 through I_5: load current of each perametron.

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