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Multiple-valued logic devices using single-electron circuits

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Multiple-valued logic devices can be constructed compactly by utilizing quantized behavior of single-electron circuits. As an example, a single-electron multiple-valued Hopfield network solving optimization problems is designed. Computer simulation shows that the network can successfully converge to its optimal state that represents the solution to the problem.

Key words: single electron, multiple valued, Hopfield network.

1. Introduction

One of the challenges in microelectronics is the development of novel electronic devices that can perform functional computing by utilizing inherent properties of quantum phenomena. We here propose one such computation device, namely a *multiple-valued logic device using single-electron circuits*.

The multiple-valued logic is a way of implementing digital operations by using a multiple set of logical values {0, 1, 2, 3, . . .} instead of a binary set {0, 1}. The logic process of the multiple-valued logic is much more sophisticated than that of binary logics, so the multiple-valued logic is expected to be more powerful for implementing digital functions with a smaller number of devices. But in practice, it has been unsuccessful to construct multiple-valued logic LSIs because most of multiple-valued logic functions are hard to implement using CMOS circuits.

To overcome this problem, we here propose an idea that multiple-valued logic systems can be constructed into a compact circuit by using the single-electron circuit technology. The single-electron circuit is a quantum electronic circuit based on the Coulomb blockade effect in electron tunneling. A conspicuous property of the single-electron circuit is that the circuit shows 'quantized behavior' in its operation; i.e. the variation of the charge on each node of a single-electron circuit is quantized in units of the elementary charge because the node charge is changed only through electron tunnelings. By utilizing this, we can construct various circuits for multiple-valued logic operations. As an example, implementation of the multiple-valued Hopfield network is discussed in the following.

2. The multiple-valued Hopfield network

The multiple-valued Hopfield network is a computation device for solving combinatorial optimization problems with multiple-valued variables. The concept of the network is illustrated in Fig. 1. The network consists of many neurons and connections. The output of each neuron feeds back into inputs of other neurons.

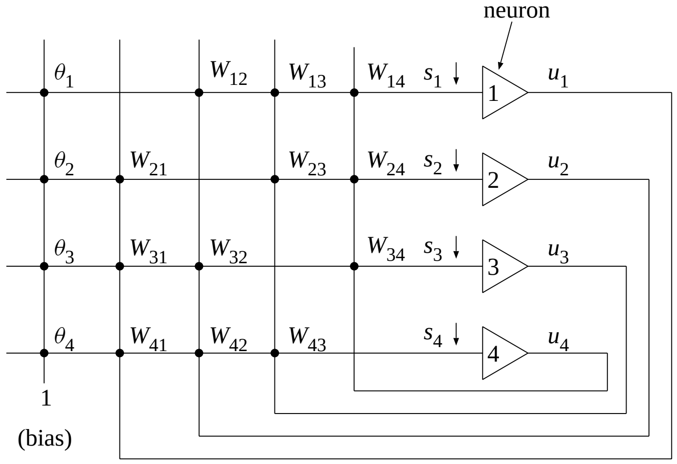

Fig. 1. Hopfield network.

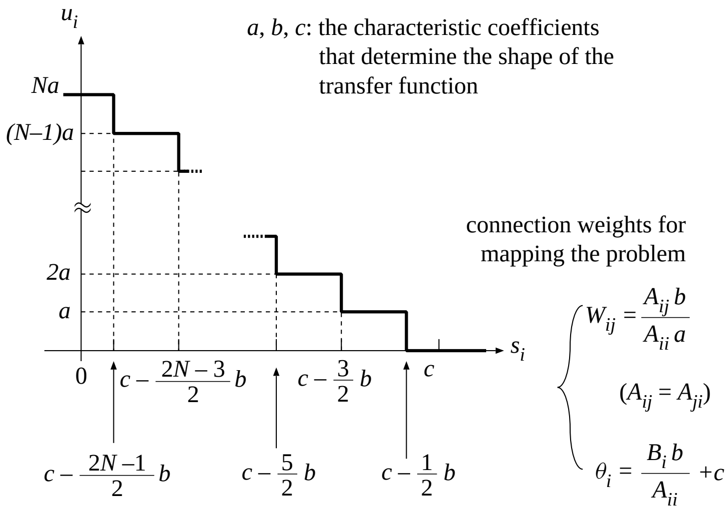

Fig. 2. Staircase transfer function.

Denoted by W_{ij} is the connection weight to neuron i from neuron j, and θ_i is the bias-connection weight to neuron i from a bias that fixed at the value of 1, and u_i is the output of neuron i. A set of neuron outputs $(u_1, u_2, \ldots)$ is called the state of the network. In this network, each neuron i takes a weighted sum s_i of its inputs ($s_i = \sum_j W_{ij}u_j + \theta_i$) and generates the corresponding output u_i according to a given staircase transfer function. All neurons operate in parallel to update their outputs continuously, and the network converges to an optimal state through the updating process.

The structure of combinatorial optimization problems can be mapped into the structure of the network by deciding the connection weights between neurons. As an example, we here take up the quadratic integer-programming problem: Given a set of coefficients A_{ij} and B_i ($A_{ij} = A_{ji}, A_{ii} \neq 0; i, j = 1, 2, 3, \ldots, n$),

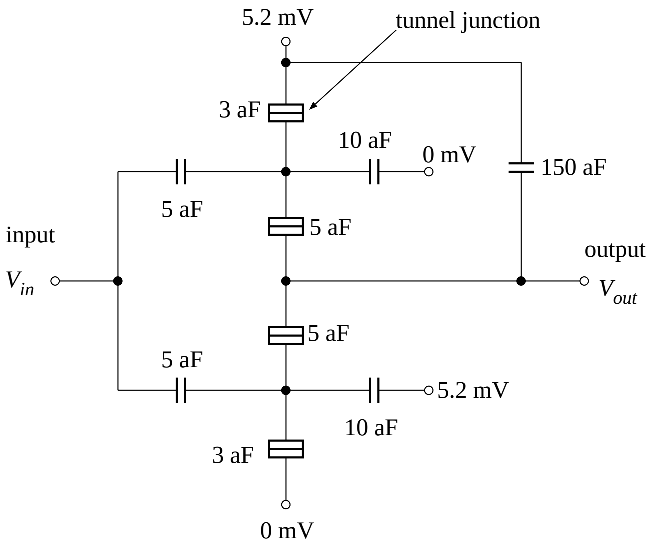

Fig. 3. Single-electron neuron circuit.

minimize objective function

$$\frac{1}{2}\sum_{i}^{n}\sum_{j}^{n} A_{ij}x_i x_j + \sum_{i}^{n} B_i x_i, \quad \text{where } x_i \in 0, 1, 2, \ldots, N \ (i = 1, 2, \ldots, n). \tag{1}$$

To solve this problem, we prepare a multiple-valued Hopfield network such that the output u_i of each neuron represents each problem variable x_i; we prepare neurons with the staircase transfer function illustrated in Fig. 2 and represent problem variables x_i by $x_i \equiv u_i/a$. The connection weights required for mapping the problem are given in Fig. 2. In problem solving, we set the network in an initial state (any state will do), then allow it to change its state without restraint. After some transition time the network converges to a final state. If convergence has been successful, the solution to the problem is obtained from the final state of the network (we will not discuss the local-minimum effect here).

3. Constructing a multiple-valued neuron using single-electron circuits

The single-electron circuit is an electronic circuit consisting of tunnel junctions and capacitors that is designed for manipulating electronic functions by controlling the transport of individual electrons. The internal state of the circuit is determined by the configuration of its electrons (i.e. the pattern in which the excess electrons are distributed among the nodes of the circuit). The circuit changes its electron configuration through electron tunnelings in response to the input and, thereby, changes its output voltage as a function of inputs.

Our purpose is to construct a single-electron neuron circuit that changes the charge on its output node (therefore its output voltage) according to a staircase function of the input voltage. For this purpose, we take *Tucker's single-electron inverter* and modify its circuit parameters to create a staircase transfer function (for the details of Tucker's original circuit, see [1]). The circuit we use for the neuron devices is illustrated in Fig. 3 together with a sample set of device parameters. The corresponding transfer characteristic are illustrated in Fig. 4. The number of steps in the transfer characteristic can be controlled by designing the circuit parameters.

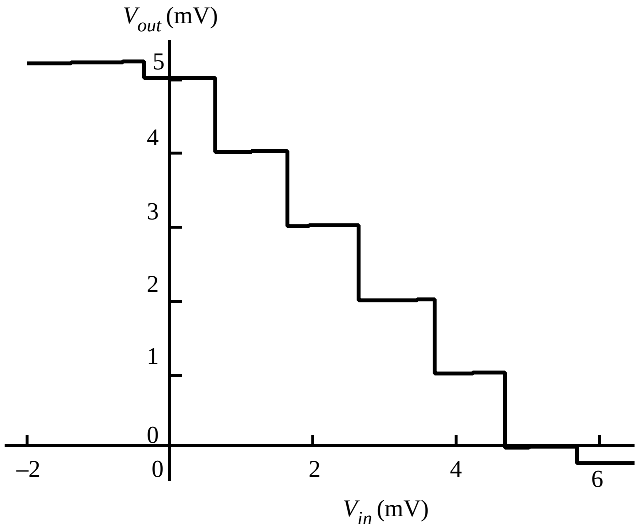

Fig. 4. Staircase transfer characteristic.

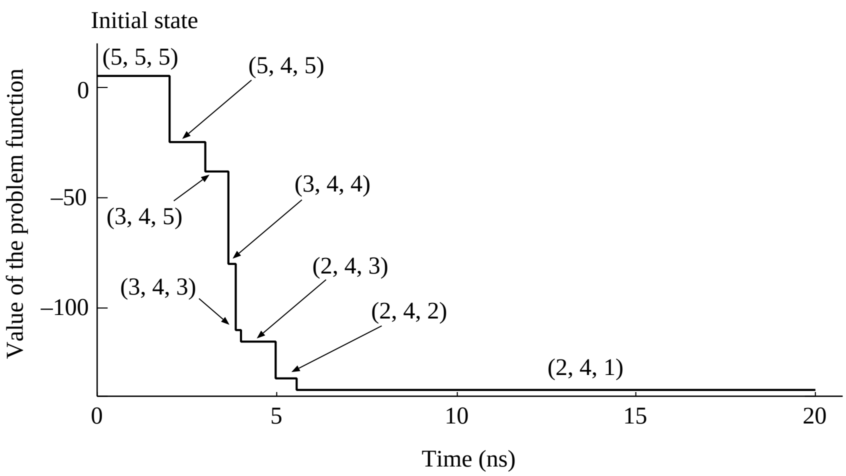

Fig. 5. State transition of the network (simulation).

4. Operation of a multiple-valued Hopfield network

The multiple-valued Hopfield network can be constructed by combining the developed neuron circuits into a network. We illustrate here a sample network that solves an instance of the quadratic integer-programming problems.

Consider the following problem:

Minimize

$$3x_1^2 + 6x_2^2 + 6x_3^2 + 4x_1x_2 - 2x_2x_3 + x_3x_1 - 29x_1 - 53x_2 - 7x_3, \qquad \text{where } x_i \in \{0, 1, 2, 3, 4, 5\}. \quad (2)$$

To solve this problem instance, we prepare a network with three neuron to represent problem variables x_i by output u_i of i-th neurons ($i = 1, 2, 3$). We used the neuron circuit with the transfer characteristic illustrated in Fig. 4 and determined the value of connections for mapping the given problem. For problem solving, it is essential that, starting with a given initial state, the network circuit should converge to its global minimum energy states. To observe the behavior of the sample network, we simulated the state transition of the network.

The results of the simulation are illustrated in Fig. 5 in which the state of the sample network is expressed by a normalized set of three neuron outputs $(u_1/a, u_2/a, u_3/a)$, where u_i/a represent problem variables x_i. The circuit was initially set at state (5, 5, 5), then it was allowed to change its state without restraint. After some transition time, the circuit stabilized in the final state (2, 4, 1) that corresponds to the solution to the problem. We repeated the same trial many times and confirmed that every trial resulted in successful convergence to state (2, 4, 1).

5. Conclusion

We proposed that multiple-valued logic devices can be constructed into a compact circuit by using single-electron circuit technology. To present a practical form of our idea, we designed a single-electron multiple-valued Hopfield network and confirmed that the network can successfully converge to its optimal state representing the solution to the problem. Our results show that multiple-valued LSIs can be fabricated by using single-electron circuit technology.

References

[1] J. R. Tucker, J. Appl. Phys. **72**, 4399 (1992).