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A single-electron circuit as a discrete dynamical system

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Abstract

A single-electron circuit can be operated as a discrete dynamical system because it changes its internal state discontinuously because of electron tunneling. To confirm this idea, we designed a sample circuit for discrete dynamical operation and confirmed by computer simulation that the circuit successfully generated a sequence of discrete-time outputs by following a return map. The concept of discrete dynamical systems will be useful in developing new functional systems that consist of quantum devices and nanostructures.

Keywords: Single-electron circuit; Return map; Chaos; Discrete dynamical system; Bifurcation

1. Introduction

The single-electron circuit is an electronic circuit designed to manipulate electronic functions by controlling the transport of individual electrons, making use of the Coulomb blockade phenomenon [1]. A distinctive characteristic of the single-electron circuit is that the circuit changes its state discontinuously because of electron tunneling. This enables single-electron circuits to be operated as discrete dynamical systems. We propose an example of such a discrete dynamical system and illustrate its operation using results obtained by computer simulation. Our goal is to create new information-processing devices that can make use of the discrete dynamical behavior of single-electron circuits.

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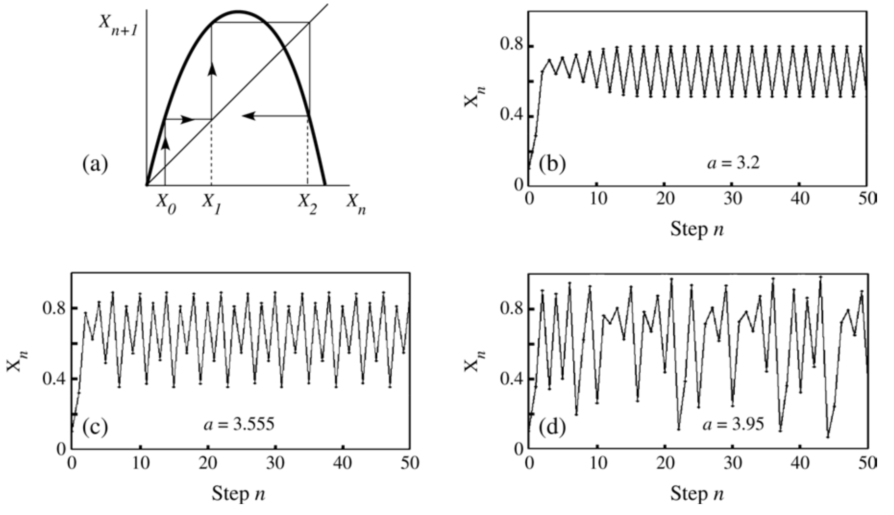

Fig. 1. The discrete logistic system expressed by $X_{n+1} = aX_n(1 - X_n)$: (a) return map, (b) two-cycle oscillation, (c) eight-cycle oscillation, and (d) chaotic behavior.

2. Discrete dynamical system

A discrete dynamical system is a system in which the evolution of the variables is measured in discrete time steps. The behavior of the system is governed by a difference equation, or a return map, that gives the $(n + 1)$th value of variables as a function of the preceding nth value of the variables.

A discrete dynamical system shows complex behavior even in simple systems with few variables [2]. An example is the *discrete logistic system*, a single-variable system whose behavior is expressed by the return map,

$$X_{n+1} = aX_n(1 - X_n),$$

shown in Fig. 1(a), where X_n and X_{n+1} are the nth and $(n + 1)$th values of the variable. In this system, a series of bifurcations occurs as the positive parameter a increases. Consequently, the system exhibits dynamical behavior from a variety of periodic oscillations to chaos. Some examples are shown in Fig. 1(b)–(d).

3. Single-electron circuits for producing discrete dynamical behavior

Because of electron tunneling, the single-electron circuit produces discontinuous change of its internal state, allowing it to be operated as a discrete dynamical system. As an example, we propose a single-electron circuit consisting of two coupled single-electron oscillators. Fig. 2 shows the configuration. In this circuit, one oscillator consists of a resistor R_1 and a *left* tunneling junction C_j, connected in series at node 1 and biased with a positive voltage V_{dd}. The other oscillator consists of a resistor R_2 and a *right* tunneling

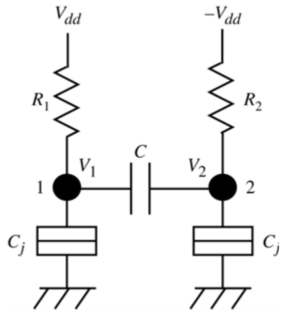

Fig. 2. A single-electron circuit consisting of two coupled oscillators.

junction C_j at node 2 and biased with a negative voltage $-V_{\mathrm{dd}}$. The oscillators are coupled by a capacitor C. The variables of the circuit are node voltages V_1 and V_2.

Without the coupling capacitor, each oscillator would produce a self-induced simple SET oscillation independent of the other. Coupled by capacitor C, the two oscillators interact with each other to produce the phenomenon of entrainment. For example, suppose electron tunneling occurs in the left oscillator through junction C_j from the ground to node 1. Then node 1 carries a negative charge to decrease its voltage to a negative value, and this may induce tunneling in the right oscillator from node 2 to the ground. Similarly, tunneling in the right oscillator may induce tunneling in the left oscillator. In this way, the two oscillators influence each other and cause a mutual relationship between node voltages V_1 and V_2.

4. Simulating discrete-time behavior of the coupled oscillators

By computer simulation, we studied the dynamics of this circuit. In the simulation, we assumed the tunneling waiting time to be zero. In other words, tunneling was assumed to occur as soon as the voltage condition for the Coulomb blockade was broken in the circuit.

Fig. 3 shows an example of the operation with the waveform of node voltage V_2 (node voltage V_1 shows a similar waveform but is not illustrated). In this operation, we define the variable of the circuit as the value of V_2 measured just before electron tunneling occurs (or just before a waveform jump occurs); the nth value of the variable is denoted by X_n in the figure. The circuit can therefore be considered as a dynamical system that produces a sequence of discrete-time variables X_n $(n = 0, 1, 2, \ldots)$. By plotting X_{n+1} as a function of X_n, we can obtain a return map that governs the dynamics of the system. To draw an exact return map, we studied the phase portrait of the system operation, as described next.

5. The phase portrait and return map for system operation

The system operation is governed by the equations in (a) and (b) below:

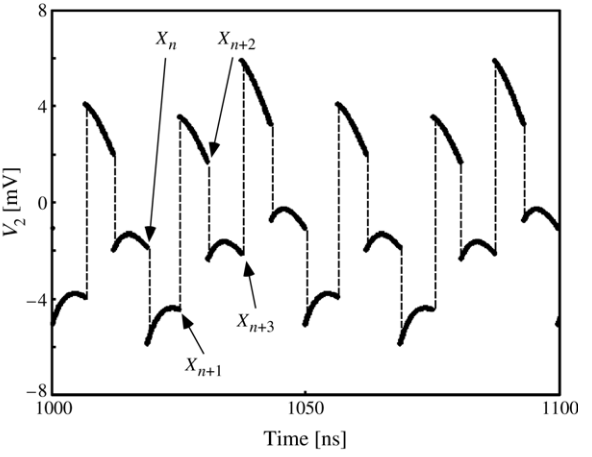

Fig. 3. The waveform of node voltage V_2 in the coupled oscillators, simulated with parameters $C_j = 10$ aF, $C = 5$ aF, $R_1 = 0.4$ GΩ, $R_2 = 1.2$ GΩ, and $V_{dd} = 12$ mV. The discrete-time variable is denoted by X_n.

(a) node voltages V_1 and V_2 change continuously in the following way:

$$\mathrm{d}V_1/\mathrm{d}t = \{R_2(1 + C/C_j)(V_{dd} - V_1) - R_1 \times (C/Cj)(V_{dd} + V_2)\}/\{CR_1R_2(1 + 2C/C_j)\}$$

and

$$\mathrm{d}V_2/\mathrm{d}t = \{R_2(C/C_j)(V_{dd} - V_1) - R_1 \times (1 + C/C_j)(V_{dd} + V_2)\}/\{CR_1R_2(1 + 2C/C_j)\}$$

if the node voltages V_1 and V_2 are in a range of $-V_{th} \leq V_1, V_2 \leq V_{th}$, where $V_{th} = e(1 + C/C_j)/\{2C(1 + 2C/C_j)\}$;

(b) when V_1 or V_2 exceeds this range, electron tunneling occurs in the circuit and this changes the values of V_1 and V_2 discontinuously in increments ΔV_1 and ΔV_2 given by

(b-1) $\Delta V_1 = -V_0$ and $\Delta V_2 = -(C/C_j)V_0/(1+C/C_j)$ when V_1 exceeds the upper threshold V_{th}, where $V_0 = e(1 + C/C_j)/\{C(1 + 2C/C_j)\}$,

(b-2) $\Delta V_1 = V_0$ and $\Delta V_2 = (C/C_j)V_0/(1+C/C_j)$ when V_1 decreases beyond the lower threshold $-V_{th}$,

(b-3) $\Delta V_1 = -(C/C_j)V_0/(1+C/C_j)$ and $\Delta V_2 = -V_0$ when V_2 exceeds the upper threshold V_{th}, and

(b-4) $\Delta V_1 = (C/C_j)V_0/(1+C/C_j)$ and $\Delta V_2 = V_0$ when V_2 decreases beyond the lower threshold $-V_{th}$.

The resulting dynamics of the system depends on the circuit parameters (C_j, C, R_1, R_2, and V_{dd}). We have not yet succeeded in giving a general, analytical expression for the return map. Instead we show an example of the return map obtained by computer simulation with a sample set of circuit parameters.

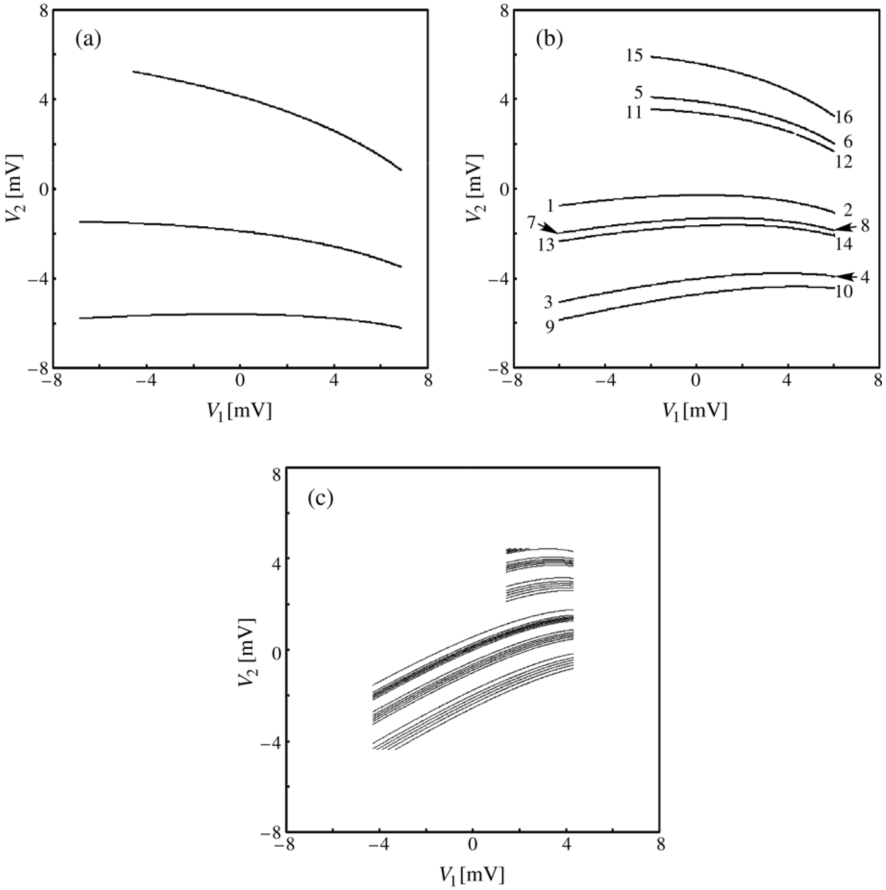

Fig. 4. The attractor of the operation plotted on a V_1–V_2 phase plane, simulated with different values for the coupling capacitance C. The other parameters are the same as in Fig. 2. The coupling capacitance C is (a) 2 aF (3-cycle oscillation), (b) 5 aF (8-cycle oscillation), and (c) 20 aF (36-cycle oscillation).

To obtain the return map, we simulated the system operation and plotted the trajectory of the oscillation on a V_1–V_2 phase plane. The trajectory depends on initial values of V_1 and V_2, but was attracted, as time passed, to a set of curves (an attractor) independent of the initial conditions. Fig. 4(a)–(c) show the attractor for three values of coupling capacitance C. The system exhibited periodic oscillation and, roughly speaking, the number of cycles increased with the coupling strength.

Fig. 4(b) explains the operation in detail. The system produces an eight-cycle oscillation. The attractor for the oscillation starts at point 1, proceeds rightward to 2, jumps discontinuously to 3 because of tunneling in the right junction, proceeds rightward to 4, jumps discontinuously to 5 because of tunneling in both junctions, and finally returns to 1. The resulting flow on the attractor is $1 \to 2 \to 3 \to \cdots \to 16 \to 1$. We simulated the system operation using different initial conditions to examine the transient trajectories

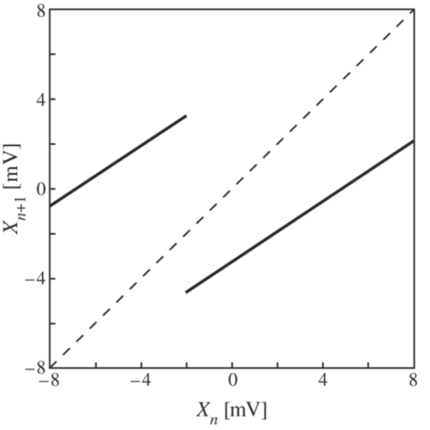

Fig. 5. The return map for the system with the parameter set shown in Figs. 3 and 4(b).

that lead to the attractor. From this phase portrait, we obtained a return map that gives the sequence of discrete-time variables X_n (the values of V_2 just before electron tunneling). This map is shown in Fig. 5 (the shape is similar to that in the Nagumo–Sato mathematical neuron model). We are now studying the general expressions for the return map to grasp the entire dynamics of the system.

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